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An Examination of the Discourse in a Graduate Mathematics Methods Course

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AN EXAMINATION OF THE DISCOURSE IN A GRADUATE MATHEMATICS METHODS COURSE

by

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DEDICATION

I dedicate this dissertation to my husband, Jay, and my three children, Mia, Jack and Liam. Their never-ending support made this journey worthwhile. I am forever grateful for their love. Each of them makes me a better person every day.

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ABSTRACT

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by

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Mathematics reform efforts advocate the use of discourse as a method toward mathematical learning. Research also suggests that attention be placed on prospective teachers' development of content knowledge and pedagogical content knowledge. One way to develop teacher knowledge and discourse skills of prospective teachers is to engage them in purposeful discourse that focuses on mathematical knowledge for teaching. The purpose of this study was to examine the discourse of prospective mathematics teachers in a graduate mathematics methods course. Theories of communication developed a framework for analyzing discourse. Theories on teacher knowledge formed a framework for analyzing the types of teacher knowledge the prospective teachers encountered. Additional theories of self-efficacy and motivation were utilized to conceptualize the factors that affected discourse. A single case study was used to examine the discourse of seven prospective teachers in their initial mathematics methods course along with their professor. Data were collected during three class meetings. Discussions were recorded, field notes gathered, documents collected, and writing prompts completed. Interviews were conducted with three prospective teachers through electronic mail. Inductive analysis was used to analyze the data. Findings indicate that teacher knowledge is separated into two entities and different characteristics exist. Mathematical discourse resembled traditional classroom discourse containing uni-directional communication, low cognitive questioning, and low levels of engagement. Pedagogical discourse exhibited contributory communication, open-ended questioning, and higher levels of engagement. Teacher knowledge during pedagogical discourse focused on general teacher pedagogy and operational issues. There were not opportunities for mathematical pedagogical knowledge development. Implications for practice include increased focus on development of discourse skills, and increased attention to the development of pedagogical content knowledge. Implications for research include the examination of expectations of prospective teachers and factors that contribute to the development of mathematical self-efficacy.

Keywords: discourse; mathematical knowledge for teaching; questioning; pedagogy; pedagogical content knowledge; prospective teacher

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CHAPTER I

INTRODUCTION

Students and teachers have required knowledge of the language of mathematics to communicate effectively. McNair (2000) asserted, "The text of a mathematics classroom discussion is as important as the written text provided by textbooks and teacher handouts" (p. 198). According to the National Council of Teachers of Mathematics (NCTM),

> Communication plays an important role in helping children construct links between their informal, intuitive notions and the abstract language and symbolism of mathematics; it also plays a key role in helping children make important connections among physical, pictorial, graphic, symbolic, verbal, and mental representations of mathematical ideas (1991, p. 26).

Sfard (2001) affirmed the importance of conversations toward the success of mathematical learning. NCTM (1989) has realized the need for all students to experience listening, reading, writing, and speaking about mathematics. Reform efforts have charged mathematics teachers with providing an instructional environment where probing questions and explaining mathematical ideas are the norm. In addition, they have charged teachers with providing students the safety and opportunity to question and explain their thought processes with others. Learning in the mathematics classroom has become social, not individual (Brendefur & Frykholm, 2000; Burton, 2002; Nathan & Knuth, 2003; Sfard, 2001; Simon, 1995; Williams & Baxter, 1996). Just as mathematics students need the tools and skills to be effective communicators, teachers need tools to cultivate productive mathematical discourse. Brendefur and Frykholm (2000) emphasized the importance of teacher educators and their role in developing mathematical communication practices among prospective teachers through their awareness of prospective teachers' conceptions of communication. Teacher education methods courses, where pedagogical skills are traditionally developed, represent the forum for the development of mathematical communication skills. In order to engage students in purposeful mathematical discourse as a vehicle for learning, prospective teachers had to engage in and recognize purposeful discourse during their training.

Equally important to prospective teachers' development of discourse skills is the development of teacher knowledge that guides the facilitation of productive discourse. The forms of teacher knowledge that prospective teachers possess has generated interest in mathematics education (Ball, 1990; Even & Tirosh, 1995; Hill, Ball, & Schilling, 2008; Hill, Rowan, & Ball, 2005; Hill, Schilling, & Ball, 2004; Shulman, 1986; Simon, 1995; Tirosh, 2000). Shulman (1986) described content knowledge, pedagogical knowledge, and curricular knowledge as necessary forms of teacher knowledge. The manner and intensity of how mathematics methods courses integrated these two forms of teacher knowledge has been debated (Nathan & Petrosino, 2003). Shulman (1986) placed equal importance on the development of all three forms of teacher knowledge. Researchers have discovered that not all prospective teachers possess strong content knowledge (Ball, 1990; Even & Tirosh, 1995; Hill et al., 2004; Hill et al., 2005; Hill et al., 2008; Nathan & Knuth, 2003; Nathan & Petrosino, 2003; Shulman, 1986; Simon, 1995; Tirosh, 2000). Furthermore, not all prospective teachers have entered classrooms with solid pedagogical knowledge and more specifically, mathematical pedagogical knowledge. The interrelations between content knowledge and pedagogical content knowledge have not been clearly established (Even, 1993; Even & Tirosh, 1995; Nathan & Petrosino, 2003)

This study examined the discourse within a mathematics methods course to determine the level of mathematical communication that existed among prospective mathematics teachers. An additional purpose of this study was to investigate the relationship between classroom discourse and the acquisition of teacher knowledge. This study sought to determine whether discourse was a method of learning for prospective teachers.

Setting: Master of Arts in Teaching program

Teachers have entered the field of education from several different paths. Some prospective teachers realize their desire to teach immediately and enter a traditional, undergraduate teacher education program. Others might not have heard the call to teach until after they had worked in another field and thus, earned their teacher certification through an alternate teacher preparation program. For other prospective teachers, their route consists of a five-year program that offers initial teacher certification along with a master’s degree. Individuals who enter a five-year program have already earned an undergraduate degree. Regardless of the academic route, a prospective teacher is not required to complete a traditional four-year undergraduate teacher education program to become a teacher.

A large university in the southeastern part of the United States developed an initial teacher certification program five years ago to provide college graduates with an opportunity to earn their teacher certification along with a master's degree. The Master of Arts in Teaching program (MAT) offers initial certification in six specialized areas. This study focused on MAT students enrolled in the mathematics certification program. These individuals entered the mathematics MAT program with a degree in mathematics, or other related fields. They desired teacher certification in secondary mathematics. The university has offered the following description of the program (See Appendix A):

> The Master of Arts in Teaching (M.A.T.) is for individuals who already possess a bachelor's degree in mathematics and who are interested in secondary (6-12) certification in mathematics. The Master of Arts degree program leads to initial certification of well-qualified teacher candidates and prepares them to be teacher-leaders. The M.A.T. program is a content-focused, standards-based program...Course work emphasizes scholarly rigor through research and engagement in a variety of field-based action research projects. Technology and multicultural considerations are infused throughout the program (Program description, ¶ 1, 2010).

The MAT program in secondary mathematics requires 48 hours of coursework. Thirty hours are devoted to the prospective teacher's professional development. These courses focus on topics such as curriculum and instruction, assessment of learning, student psychology, action research and teaching methods. Embedded within the teaching methods courses are hours that are spent in a field experience at a local area school. In this field experience, prospective teachers observe a classroom teacher and begin teaching lessons to students. Eighteen hours are devoted to mathematics content including geometry, statistics, and set theory. The initial mathematics methods course to served as the backdrop of this study.

Traditionally, prospective teachers have viewed mathematics methods courses as the climax of teacher education coursework. In the methods courses, they have learned pedagogical theory and skills necessary to intertwine pedagogical knowledge with content knowledge. Prospective teachers have held the expectation that methods courses will aid in the development of strategies and tools for the classroom (Graeber, 1999; Wilson & Ball, 1996). Methods courses have developed prospective teachers' *how-to* skills. The catalog description of the mathematics methods course (See Appendix A) explained the purpose of the class as one that combines content and pedagogy with an examination and application of curriculum issues, learning theories, teaching strategies, instructional materials and assessment procedures for teaching middle and secondary school mathematics. In the MAT program, prospective teachers complete two mathematics methods courses, each with an accompanying field experience.

Rationale

Mathematics reform efforts have placed greater emphasis on students' abilities to engage effectively in mathematical discourse (Blanton, Berenson, & Norwood, 2001; NCTM, 1989; Wilson & Ball, 1996). While there has been extensive literature written about the mathematical discourse between the teacher and student within K-12 classrooms, less research has focused on the discourse that occurs in post-secondary classrooms and even more specifically, within the mathematics courses of prospective teachers (Ball, 1990; Cobb, Wood, & Yackel, 1992). NCTM (1989) recognized mathematics as a form of communication stating,

> The mathematics curriculum should include the continued development of language and symbolism to communicate mathematical ideas so that all students can:
>
> 1. reflect upon and clarify their thinking about mathematical ideas and relationships;
> 2. formulate mathematical definitions and express generalizations discovered through investigations;
> 3. express mathematical ideas with clarity;
> 4. ask clarifying and extending questions related to mathematics they have read or heard about (p. 140).

In order to accomplish the goals set forth by NCTM, prospective teachers need opportunities to use mathematics as a means of communication. As Nathan and Knuth (2003) explained, “it is one thing to notice that students are speaking up more in the classroom. But there is also a need to examine the nature of teacher speech and what it may say about teacher participation and mathematical practices” (p. 182). Sfard (2001) asserted that putting communication in the heart of mathematics education is likely to change the way teachers teach mathematics and think about learning. Through active student participation, teachers have the potential to direct instruction away from the focus of recall of terminology and procedures toward deeper conceptual understanding of mathematics. Teachers will have opportunities to reflect on their teaching practices through the voices of their students.

In order to promote effective mathematical discussions, prospective teachers require the development of discourse skills. Since mathematics reform efforts began, teacher methods courses have been under greater scrutiny regarding their development of meaningful pedagogical skills (Ball, 1990; Blanton et al., 2001; Cooney, 1994; Hill et al., 2004). Reform efforts have changed the expectations of today's teachers. Conventional thoughts about mathematics teaching viewed teaching primarily as the transmission of knowledge from the teacher to the student (NCTM, 2009). Ball (1990) discovered that prospective teachers viewed mathematics as rule-bound. She found that they lacked the knowledge and skills to provide explanations to students and foster explanations from students. Graeber (1999) indicated that reform efforts "suggest a classroom where teachers do less talking and more listening to students' responses and to students' discussions with one another" (p. 201).

Wilson and Ball (1996) acknowledged that the classrooms prospective teachers are being prepared for is different from the one they experienced as students. Nathan and Knuth (2003) noted that challenges of enacting classroom practices that promote discourse and discourse-based activities exist. These challenges include changing beliefs about mathematics learning and accepting teaching methods that make students, not the teacher, the primary focus in the classroom. Nathan and Knuth attributed these challenges to the disparity between the practices that prospective teachers experienced during training and the practices they participated in as students.

In reform-based mathematics classrooms, communication has helped to define the role of the teacher. Blanton et al. (2001) declared that a teacher's developing practice is deeply connected to classroom discourse. Wilson and Ball (1996) noted that prospective teachers often assume that they possess the skills to foster discourse. However, they discovered that many prospective teachers were not equipped with these skills. Wilson and Ball found that the prospective teachers in their study viewed teachers as people who cleared up errors and were the main source of information. Mathematical discussions revolve around disseminating information to students and answering questions (Cobb et al., 1992; Quinn, 1997; Wilson & Ball, 1996). Sfard (2001) affirmed this perspective. She noted that in traditional classroom discourse, there is usually a dominant authority who informs all others of the rules. This role normally belongs to the teacher.

In order to change prospective teachers' perspective on the purpose of classroom discourse, teacher education programs were charged with immersing them in purposeful discourse. Wilson and Ball (1996) each videotaped their own teaching and questioning of students so that their prospective teachers could scrutinize and discuss the discourse that occurred. According to Wilson and Ball, "The point…is to move the discourse of teacher education classes past issues of liking or not liking a particular child or lesson, to probing what is being taught and learned" (p. 130). Consequently, their teacher education methods courses became an atmosphere for not only gaining skills for practice, but also for analyzing practice.

Graeber (1999) stressed the importance of effective discourse skills as part of a teacher's pedagogy. Graeber contended that understanding and supporting students' reasoning is important to successful instruction. In order to foster student discussions, prospective teachers had to engage in the same meaningful conversations. She suggested reading about and discussing a number of prominent mathematical misconceptions. She also suggested that prospective teachers experience meaningful tasks as learners as a way to engage in purposeful discussions about their implementation in the classroom. Blanton et al. (2001) also suggested engaging prospective teachers in such conversations and then analyzing their own discussions as a method of developing discourse skills.

Marzano (2006) asserted that individuals retain 50% of what they learn by seeing and hearing. Furthermore, individuals retain and apply 90% of what they learn by doing. Therefore, it is reasonable to conclude that the experience and practice of discourse could lead to its learning. If mathematics reform efforts have advocated purposeful discourse in classrooms as a method of learning, then prospective teachers needed to experience, recognize and practice purposeful mathematical discourse. They need to develop the skills that allow them discern when to aid in knowledge construction and when to allow students to construct their own knowledge (Cobb, Boufi, McClain, & Whitenack, 1997; NCTM, 1991; Simon, 1995; Williams & Baxter, 1996). These skills include knowing when to encourage discovery and when to monitor and scaffold ideas.

Nathan & Knuth (2003) stressed that “teacher education and professional development programs need to…provide preservice and practicing teachers with both the experience participating in and the tools to facilitate productive mathematical discourse” (p. 204). Teacher educators possess the opportunity to provide prospective teachers with strong mathematical content knowledge intertwined with sound pedagogical skills using purposeful discourse. Davis (1997) claimed that teacher education should not focus on subject matter, teaching methods or theories. He suggested that prospective teachers engage in discussions of the big ideas that surround mathematics and mathematics pedagogy. Through this discourse, integrated learning of mathematics content and mathematics pedagogy has the potential to develop and flourish.

Statement of Purpose of Research and Research Questions

The purpose of this study was to analyze the classroom discourse of a mathematics methods course. By analyzing the discourse of the prospective teachers in the course, I assumed that the existence of a relationship between the discourse and the acquisition of mathematical content and pedagogical knowledge could be determined. Additionally, an examination of prospective teachers' discussions has the potential to provide insight into the topics prospective teachers discussed and the information they internalized. This study centered on investigating and answering the following questions regarding the discourse in a graduate mathematics methods course:

1. How does discourse appear within the instructional environment of a mathematics methods course for prospective teachers?
    a. What are the patterns, functions, and nature of classroom discourse?
    b. What role does questioning play in shaping classroom discourse?
2. How did the discourse in the mathematics methods course facilitate or hinder opportunities for prospective teachers to develop mathematical knowledge for teaching?

Research Design Overview

A case study design provided a detailed, descriptive analysis of the characteristics of classroom discourse in a university setting. I submitted and obtained Institutional Review Board (IRB) approval from the university. The participants selected were prospective MAT teachers and their methods course professor. Seven prospective teachers enrolled in the mathematics methods course. I explained my purpose, intent, and all consented to be participants. The unit of analysis of the study consisted of the class rather than the individual participants. Due to my desire to have my study result in practical implications, I applied a pragmatist paradigm to the study. With a pragmatist paradigm, the focus became the outcome of the research. It searches for meaningful application of findings to real-world contexts and situations (Creswell, 2007). Thus, the study focused on the data generated and the interpretations made from the data.

Data collection on the classroom discourse of the mathematics methods class occurred over three class sessions. The prospective teachers attended class each Monday night for close to three hours. I collected multiple types of data. These included artifacts of classroom tasks, field notes of classroom observations, audio recordings of classroom discourse, interview responses, and written prompts that the prospective teachers completed after each observed class. I observed the class to gather field notes and recorded the discussions using a digital recording device. The prospective teachers completed a writing prompt anonymously after each observed class through an online survey website, Survey Monkey. Three prospective teachers participated in an interview after the course had officially ended through email correspondence. I used three instruments to collect data: a classroom observation protocol, a writing prompt, and an interview protocol containing six structured questions and four follow-up questions.

Data analysis occurred in two different ways using inductive analysis. Initially, coding of audio recording transcripts and field notes occurred by hand using a discourse coding scheme developed by Nathan and Knuth (2003) and a questioning coding scheme I created based on Anderson, Krathwohl, Airasain, Cruikshank, Mayer, Pintrich, Raths, and Wittrock’s (2001) taxonomy of cognitive processes. During the analysis, I decided to incorporate technology. I open coded my data with NVivo software using free nodes established by my coding schemes.

During axial coding, I created parent nodes that combined free nodes with similar concepts. From axial coding, I used selective coding wherein I grouped the parent nodes established in axial coding into core concepts. During selective coding, core categories led to emergent themes such as the dominance of vertical discourse and participant engagement. I tested these themes against the data. I tabulated numerical data, from the data collected on the participants’ type of verbal interactions and questioning. The numerical data supported the themes from the study’s findings.

Efforts were made to ensure the dependability and credibility of the study. To ensure dependability, the collection of data included using multiple sources. These sources included field notes, audio recordings, interview responses, and written prompt responses. Additionally, the creation of a case study protocol and the establishment a chain of evidence aided the study's dependability.

The triangulation of data sources ensured the trustworthiness of the findings. Member checking aided in the credibility of the study's findings. Member checking of the findings occurred with the professor of the course. In addition, I explored alternate explanations to the findings within the data. Research supported the efforts made to ensure both credibility and dependability (Creswell, 2007; Marshall & Rossman, 1995; Merriam, 2009; Miles & Huberman, 1994; Patton, 2002; Yin, 2003). Table 1 provides an overview of the investigation of the study.

Table 1

Overview of Investigation

Research Question	Data Sources	Type of Analysis
How does discourse appear within the instructional environment of a mathematics methods course for prospective teachers?	Field notes of classroom Observations Audio recordings Written prompts Interviews	Open coding of data Axial coding of concepts Selective coding of categories Tabulation of verbal statements of participants
What role does questioning play in shaping classroom discourse?	Field notes of classroom Observations Audio recordings Document analysis	Open coding of data Axial coding of concepts Selective coding of core categories Question frequency of participants
How did the discourse in the mathematics methods course facilitate or hinder opportunities for prospective teachers' to develop mathematical knowledge for teaching?	Field notes of classroom Observations Audio recordings Document analysis Written prompts Interviews	Open coding of data Axial coding of concepts Selective coding of core categories

Limitations and Delimitations

The study contained a small sample size, which was a limitation of the study. The sample consisted of seven prospective teachers and one professor. Thus, I could not conclude that the results of this study were applicable to all methods courses or to all prospective mathematics teachers. However, the small sample did allow me to provide insight into the nature of discourse for a select group of prospective teachers in a specialized teacher certification program.

An additional limitation included my acquaintance with some of the participants. One of the prospective teachers was completing her field experience at the school in which I was an administrator. She worked under a mathematics teacher in my grade level, which made me her supervisor. This could have limited the comments and questions she asked during class discussions. Likewise, another prospective teacher worked at the same school as my husband. Therefore, that relationship could have affected her participation and selection of statements. She may have assumed that I would share her statements in class with my husband.

Finally, data collection was a limitation to the study. Due to my work schedule, I could not observe the prospective teachers within their field experience. Being able to conduct those observations had the potential to determine a correlation between the prospective teachers as participants in discourse and as facilitators of discourse. Additionally, data collection did not occur for all of the classes because IRB approval did not happen until the middle of the semester. Therefore, the topic and characteristics of those classes remained unknown. I could not determine whether the discourse of previous classes had an impact on the discourse of the observed classes.

Because I wanted to gain understanding of the prospective teachers’ discourse on mathematics teaching and learning, some delimitations were necessary. First, the prospective teachers selected were all part of the MAT program rather than a mixture of graduate and undergraduate prospective teachers. Second, the prospective teachers selected were in the mathematics cohort rather than across other disciplines. I selected mathematics and a course that focused on pedagogy within a mathematical context due to my previous work experience as a middle school mathematics teacher and as a professional learning lead teacher.

As a mathematics teacher, my priority had been to provide students with the opportunity to view mathematics as something other than a set of rules and algorithms. Additionally, I had always maintained an interest in promoting student engagement in the mathematics classroom. I felt the mathematics methods course would be an ideal setting in which I could gain further knowledge of current research and strategies on mathematics engagement through the course’s focus on mathematics content and pedagogy. Third, due to my work schedule, the selected participants attended class at night, which enabled me to conduct classroom observations without impeding on my work obligations.

Propositions

Yin (2003) defined a proposition as a statement that declares the researcher's assumptions about the study. At the outset of this study, I made three propositions. My first proposition was that the discourse among prospective teachers would occur equally between peers and with the professor. I assumed that the prospective teachers, being adult learners, would be equipped with the skills to engage in meaningful discussions and self-reflection on their own views of mathematics teaching from those discussions. I also assumed that their discourse would result in an altering of thinking or practice.

My second proposition was that an unequal distribution of time would exist between discussions involving the development mathematical content and mathematical pedagogical knowledge. I assumed that the prospective teachers in this course were more interested in learning about mathematics pedagogy rather than mathematics content. Their interest in mathematics pedagogy would affect the focus of their conversations. Their assumed interest in mathematics pedagogy rather than content could be because the prospective teachers in the course already held a degree in mathematics or a mathematics-related field. It could exist because the unknown for prospective teachers is the *how* of teaching more than the *what* of teaching. Furthermore, the course was a methods course and not a mathematics content course. Therefore, this implied that mathematics content was a secondary focus.

My final proposition was that the prospective teachers' purpose for participating in discourse would be to gain or clarify pedagogical knowledge. This assumption relates to the previous proposition. Prospective teachers may have assumed that the purpose of mathematics methods courses was learning to teach as opposed to learning about mathematics. This assumption can lead to the desire of gaining tangible teaching strategies. I assumed that prospective teachers viewed mathematics content and mathematics pedagogy as two separate entities. I assumed that they would use discourse as a means of sharing their classroom experiences. Additionally, discourse served as a way to clarify their responses to situations that occur in the classroom.

Definitions of Terms

The terms defined for this study are terms that may contain more than one definition depending on the use of the term and the context it is used. Therefore, the definitions provided below can add clarity for the reader when encountering these terms throughout the study.

Discourse

As defined by Brendefur and Frykholm (2000), discourse is the communication of thoughts and ideas through words either written or verbal. Discourse can occur in two ways. Hatano and Inagaki (1991) defined vertical discourse as a verbal interaction between an individual viewed to possess more knowledge on a topic and another individual viewed to possess less knowledge. They defined horizontal discourse as a verbal interaction between individuals who are viewed to possess the same amount of knowledge about a topic.

Prospective teacher

Prospective teacher is defined as a student within a teacher education program who is seeking initial teacher certification. A prospective teacher could be an individual who did possess a certificate or held a non-renewable certificate. For this study, a prospective teacher was a student enrolled in the mathematics methods course.

Scaffolding

Scaffolding, as defined by Williams and Baxter (1996) occurs when information or ideas is provided by someone with knowledge to someone without knowledge in order to support learning. They proceeded to define *analytical scaffolding* as scaffolding that occurs for an instructional topic rather than for social norms or behaviors.

Pedagogy

Shulman (1986) defined pedagogy as the knowledge, including skills and strategies, of teaching. He sub-divided pedagogy into categories including general teacher pedagogical knowledge, and content-specific pedagogical knowledge.

Overview of Study

In the next four chapters, I outline, present, and discuss my study and its findings concerning the classroom discourse within a graduate level mathematics methods course. Chapter two provides a review of the literature related to mathematical discourse, questioning, and mathematics for teaching along with the theoretical framework that grounded my study. Chapter three outlines the methodology used to carry out the study including data sources and analysis. In Chapter four, I present my findings in terms of significant themes that resulted from the analysis of the data. Chapter five discusses my theoretical analysis of the findings and provides answers to my research questions. Chapter six concludes with a summary of the study and the implications my study has for future research and practice in mathematics education.

CHAPTER II

REVIEW OF RELEVANT LITERATURE

Topical Review of Relevant Literature

Mathematical Discourse in the Classroom

The influx of research regarding mathematical discourse has shown that mathematics education researchers have become aware of the importance of mathematical conversations for successful student learning (Blanton et al., 2001; Brendefur & Frykholm, 2000; Cobb et al., 1992, 1997; Lampert, 1990; Nathan & Knuth, 2003; Sfard, 2001; Simon, 1995; Williams & Baxter, 1996). Communication has become a focal point of mathematics reform because it is through meaningful conversations about mathematics that students construct meaning (Simon, 1995). NCTM (1991) affirmed the role of discourse stating, “The nature of classroom discourse is a major influence on what students learn about mathematics” (p. 45). Cobb et al. (1992) supported the power of mathematical discourse as an avenue that results in shared mathematical knowledge. Simon (1995) acknowledged the importance of discourse in the development of students and teachers’ mathematical content knowledge. Blanton et al. (2001) also predicted a continued emphasis on mathematical discourse stating, “discourse informs not only our understanding of students’ thinking about mathematics, but also teachers’ thinking about teaching mathematics” (p.227). McNair (2000) acknowledged that the “mathematics classroom should reflect an intentional effort to learn about a concept or procedure that has become problematic” (p. 199).

Mathematics education literature has presented characteristics that describe purposeful mathematical discourse. Discourse literature has described purposeful discourse as constructive because students construct learning from what others think (Ball, 1990; Nathan & Knuth, 2003; Simon, 1995). Cobb et al. (1997) described the joint student construction of knowledge as resulting in collective reflection. Lampert (1990) described how mathematical discourse allows collaborative knowledge construction to occur among students rather than individually between the teacher and student.

Discourse literature has described effective mathematical discourse as being student-focused. Traditional classroom discourse incorporates limited interactions among students. Furthermore, it is the teacher leading the discussion, not the students. Williams and Baxter (1996) stated that discourse “is a focus on the student or on groups of students, as the seat of knowledge production, with the teacher seen as facilitating the creation of useful and appropriate knowledge” (p. 22). They noted that while students take a primary, dominant role in mathematical discussions, the teacher continues to be an active participant.

NCTM (1991) has described the role of a teacher engaged in facilitating purposeful mathematical discourse as one who can filter and direct students’ explorations of concepts. Cobb et al. (1997) stressed the importance of the teacher’s role in student discourse. They described the teacher as a guide who initiates shifts in the flow and topics of the discourse. “Initiating and guiding the development of reflective discourse requires considerable wisdom and judgment on the teacher’s part” (Cobb et al., 1997, p. 269). The teacher may not have been the transmitter of knowledge at all times, but still played a crucial role in ensuring that students came to the correct understandings.

Much of the literature on mathematical classroom discourse has depicted classrooms in which discourse is predominantly vertical occurring between teacher and student and is teacher-centered (Blanton et al., 2001; Ball, 1990; Brendefur & Frykholm, 2000; Cobb et al., 1997; Nathan & Knuth, 2003; Williams & Baxter, 1996). Brendefur and Frykholm (2000) noted that either the teacher or the student can instigate vertical discourse and it flows in both directions. Sinclair and Coulthard (1975) discovered that traditional classroom discourse follows a consistent pattern. They found that it typically follows an Initiation-Response-Evaluation (IRE) pattern. In an IRE pattern, the teacher asks a question, a student responds to the question, and the teacher evaluates the response. Blanton et al. (2001) analyzed the discourse of a prospective mathematics teacher in order to determine the nature of her discourse and developing practice. They found that the nature of the prospective teacher's discourse followed the IRE pattern. They noted that the prospective teacher asked a question to the class, a student provided a response, and the teacher either affirmed the answer if correct, provided a hint to lead the student to the correct answer or asked another student for the answer.

Nathan and Knuth (2003) followed a middle school teacher over a two-year period as she learned about the benefits purposeful discourse had the potential to provide. The teacher discovered that the discourse in her mathematics classroom was overwhelmingly vertical in nature with little student-to-student interaction. Furthermore, through the researchers' prompting for self-reflection, the teacher mediated her practice to include more opportunities for students to engage in verbal interactions with each other. Thus, she shifted her idea of classroom roles from the teacher as the teller to the students as active participants in learning. While horizontal (student-to-student) discourse allows students to conjecture, refute and use each other's ideas as objects of discussion, the teacher remains an active participant to ensure that discussions are leading to mathematical understanding (Blanton et al., 2001; Cobb et al., 1997; Nathan & Knuth, 2003).

Researchers have noted teacher listening as being an essential skill in the facilitation of effective classroom discourse (Davis, 1997; Nathan & Knuth, 2003). Davis (1997) asserted the importance of listening in discourse-oriented classrooms stating, "an attentiveness to how mathematics teachers listen may be a worthwhile route to pursue as we seek to understand and consequently, to help teachers better understand their practice" (p. 356). He also noted that effective teacher listening enables effective teacher questioning and the facilitation of student discourse of mathematical concepts. Listening allows teachers to determine when discourse is effective and when teacher guidance is necessary.

Teachers have experienced challenges in facilitating purposeful discourse (Blanton et al., 2001; Ball, 1990; Brendefur & Frykholm, 2000; Cobb et al., 1997; Nathan & Knuth, 2003; Williams & Baxter, 1996). For teachers, "enacting classroom practices that support discourse-based mathematical tasks poses difficult challenges…as such practices often bear little resemblance to teachers' current practices, or to the practices in which teachers participated as students themselves" (Nathan & Knuth, 2003, p. 176). According to Blanton et al. (2001), in order to change the nature of classroom discourse, teachers have had to confront existing classroom norms. Teachers, both in-service and prospective, have not been prepared to facilitate mathematical discussions (Nathan & Knuth, 2003). Williams and Baxter (1996) identified the fine line that exists between providing too much and too little structure that challenged teachers. Teachers have not been prepared for knowing when to step into a student discussion, when to step out or when to allow students to struggle. Teacher education programs have not focused on these discourse skills. Furthermore, teachers have struggled with the advantages and constraints mathematical discourse presented for student understanding. Therefore, teachers have gingerly incorporated classroom discourse as a vehicle for student learning.

Studies have determined that mathematical discourse promotes student learning (Brendefur & Frykholm, 2000; Blanton et al., 2001; Cobb et al., 1997; Lampert, 1990; Nathan & Knuth, 2003; Sfard, 2001; Simon, 1995). Cobb et al. (1997) cautioned that discourse can enhance students' mathematical understanding, but it cannot determine it. Through student-centered tasks in which students talk to each other about mathematics, students have the potential to come to a conceptual understanding of mathematical concepts (Brendefur & Frykholm, 2000; Cobb et al., 1997; Nathan & Knuth, 2003; Simon, 1995; Williams & Baxter, 1996). Burton (2002) discovered that the potential for mathematical understanding through discourse was not limited to student learners. He found that adult mathematicians realized the value of collaboratively constructing knowledge. The mathematicians noted advantages to their mathematical discussions such as "the increase of quantity and quality of ideas, the benefit from a novice/expert combination, and the benefit from the experience of others" (p. 162).

As Cobb et al. (1997) stated, facilitating purposeful and effective mathematical discourse requires that teachers have an understanding themselves of what classroom discourse should entail. Hufferd-Ackles, Fuson, and Sherin (2004) identified four key features of effective mathematical discourse: a) questioning, b) explaining mathematical thinking, c) source of mathematical ideas, and d) responsibility for learning (p. 91). Often, what a teacher believes she is doing is quite different from what she actually does (Blanton et al., 2001; Nathan & Knuth, 2003). In Nathan and Knuth's (2003) examination of a mathematics teacher, the teacher believed she was fostering student interaction and discussion in her classroom. However, data showed that in one year, student-to-student discussion only occurred five times.

Wilson and Ball (1996) asserted that understanding classroom discourse requires that teachers experience discourse. Prospective teachers need to experience classroom situations as learners in order to engage in discourse about the mathematics involved (Blanton et al., 2001; Wilson & Ball, 1996). Ball (1990) noted that the development of discourse skills requires that teachers have strong knowledge of mathematics and about mathematics. Furthermore, their development of how to use this knowledge in classroom situations is required.

Questioning

Pierson (2008) distinguished discourse as the primary means of instruction in the classroom. The talk that occurs in the classroom has the potential to shape student learning (Blanton et al., 2001; Brendefur & Frykholm, 2000; Cobb et al., 1992, 1997; Lampert, 1990; Nathan & Knuth, 2003; Sfard, 2001; Simon, 1995; Williams & Baxter, 1996). King (1990) found that questions often initiate conversations that take place between the teacher and students and among students. Research has long linked questioning with student learning (Bissell & Lemons, 2006; Ciardiello, 1993; Carlsen, 1991; King, 1990; Krathwohl, 2002; Mitchell, 1994; Ramsey, Gabbard, Clawson, Lee, & Henson, 1990; Redfield & Rousseau, 1981; Reeves, 2007; Sitko & Slemon, 1982; van

Zee & Minstrell, 1997; Wimer, Ridenour, Thomas & Place, 2001). Traditional teacher questioning has focused on what students know rather than what they think (Ciardiello, 1993; King, 1990; van Zee & Minstrell, 1997; Wimer et al., 2001). King (1990) noted the benefits of varying questions to achieve higher student performance. Reeves (2007) found that purposeful questioning serves as a powerful assessment tool for teachers to gauge student understanding of concepts. Reform efforts toward student engagement and student-centered discourse defined the primary purpose of questioning to be the promotion of thinking. Therefore, questioning has played a pivotal role in the shape and context of discourse. Mitchell (1994) discovered that teachers believed questioning served three purposes-managerial, instructional, and social. Furthermore, teachers viewed questioning as a vehicle to communicate knowledge and aid in the social scaffolding of the classroom.

Research has shown that teachers ask low-level questions more frequently than high-level questions (Ciardiello, 1993; Mitchell, 1994; Ramsey et al., 1990; Wimer et al., 2001). Ramsey et al. remarked, “Most of the questions asked in a typical classroom require only recitation of memorized material and are focused on the lowest levels of cognition” (1990, p. 420). In a study of gender differences regarding responses to higher order questions, Wimer et al. (2001) found that less than 25% of the questions teachers asked to students were high-level questions. They calculated that of the 249 questions asked during their observations, only 42 of the questions were higher-level questions. They surmised that limiting students’ exposure to higher-level questions limits their formation of higher-level thinking. They discovered that teachers’ knowledge of higher-level questioning was limited. Morrone, Harkness, D’Ambrosio, and Caulfield (2004) discovered in their study of prospective elementary teachers that the prospective teachers' questions that fostered higher-order thinking consisted of only two percent of all questions asked. They also discovered that the prospective teachers' questioning asked students for recall of information.

Sitko and Slemon (1982) discovered in their study that teacher questions were predominantly low-level. However, after receiving targeted training on developing higher-level questions, the percentage of higher-level questions increased. The number of questions that asked students to apply and critique increased, as did the variety of questions asked. They also found a close relationship between the level of teacher questions and the level of student responses. Their finding supported the causal relationship that a low-level question yields a low-level response.

In analyzing instructional tasks and discourse in second-grade mathematics classes, Hiebert and Wearne (1993) discovered that teachers asked more recall questions than any other type of questions. Additionally, they found a strong, statistically significant correlation between the frequency of lower-level questions, the number of problems students worked on and the amount of time students spent completing seatwork. A larger amount of time spent on seatwork corresponded to an increase in the frequency that students encountered lower-level questions. Brendefur and Frykholm (2000) asserted that teachers need training to develop questioning skills that go beyond determining recall of information.

Higher-level questioning leads to higher levels of cognition (Fairbairn, 1987; Hiebert & Wearne, 1993; Mitchell, 1994; Pierson, 2008; Sitko & Slemon, 1982; van Zee & Minstrell, 1997). Redfield and Rousseau (1981) performed a meta-analysis on 20 studies regarding questioning. Their review of these studies resulted in a positive correlation between higher-level questioning and student achievement. NCTM (2009) strongly advocated the integration of higher-level questioning to spark deeper conceptual understanding and application of mathematical concepts. While research findings were consistent regarding the value of higher-level questioning, Hiebert and Wearne (1993) cautioned that variations could exist regarding the processes and procedures surrounding the development and implementation of this type of questioning. Teacher beliefs, time constraints, and standardized testing have prompted teachers to seek affirmation of shallow learning rather than investing in deeper conceptual understanding that require more work and time. Mitchell (1994) found that teacher beliefs regarding student learning affected the level of cognitive questions teachers asked. Teachers who possessed high expectations for learning and believed in student discovery asked fewer questions. However, these teachers asked questions that evoked higher-level thinking.

van Zee and Minstrell (1997) studied the effects on student learning and behavior when Minstrell integrated higher-level questioning into his instruction. Their study focused on the learning outcomes that occurred when students encountered a *reflective toss*. They described a reflective toss as an interaction where a student makes a statement; the teacher catches the statement, and then throws the responsibility for the statement back to the student. Once the responsibility is thrown back to the student, the student provides an explanation for the statement and then the class collectively can engage in a discussion. The move back to the student causes the student to elaborate on her own thinking. They discovered that engaging in reflective tosses allowed students to construct representations of meaning, clarify understandings, and make evaluative judgments.

Bissell and Lemons (2006) studied the benefits of consciously and strategically implementing higher-level questions into a college-level Biology course. They discovered that by constructing and assessing the critical value of questions before student interaction with the questions, teachers promoted higher-level critical thinking in students. The students became aware of the type of questions asked, so they also became aware of the expectations for the responses. Planning the level and type of questions beforehand allowed the teachers to promote specifically the level of critical thinking they desired.

Engagement and Motivation of Prospective Teachers

Mathematics reform efforts have placed emphasis on student engagement in mathematics (NCTM, 1991, 2000). Researchers have used motivation and self-efficacy theory to define engagement (Bandura, 1993, 1997; Eccles & Wigfield, 2002). Bandura (1993) directly linked self-efficacy beliefs with motivation. He asserted that individuals are motivated to accomplish tasks based on their beliefs about their ability, their interest in the task, and the expected outcomes of the task. Bandura categorized motivation based on three factors: casual attributions, outcome expectancies, and cognized goals. He found casual attributions possess a correlation with self-efficacy because individuals who are highly self-efficacious attribute failure to insufficient effort. They believe that it is not their cognitive ability that prevents task completion, but low effort. Individuals who possess low self-efficacy attribute failure to low ability. They are already unconfident in their cognitive abilities; therefore, it is their ability and not their effort that prevents task completion.

Jansen (2008) discovered in her study of prospective teachers and their motivating factors toward participating in whole-class discussions that the prospective teachers who did not participate in discussions described themselves as having low mathematical self-efficacy. They were not comfortable in their mathematical abilities and therefore, did not want to participate and risk being seen as and feeling less intelligent than their peers. Their feelings about their own mathematical ability affected the way that they interacted with their peers. Mewborn (1999) found that prospective teachers possessed lower feelings of self-efficacy toward classroom life, which made this their top area of concern and motivation. She discovered that they were most curious about issues such as the physical arrangement of the classroom and classroom management. Prospective teachers next valued the development of teaching skills. She also found that prospective teachers were the least interested in developing their mathematical content knowledge.

With outcome expectancies, the expectation that behavior will produce certain outcomes drives motivation (Bandura, 1993, 1997; Eccles & Wigfield, 2002). Eccles and Wigfield (2002) noted that individuals place values on these outcomes. Bandura (1997) noted that self-efficacy plays a determining role in outcome expectancies because an individual who is not self-efficacious might not pursue a task that has a positive outcome expectancy.

In Jansen's (2008) study of prospective teachers during whole-class discussions, those prospective teachers who participated in mathematical discussions possessed the expectation that participation would help them learn mathematics and prepare them for their future career as a mathematics teacher. Korthagen and Kessels (1999) discovered that a contributing factor to the disconnect between teacher education programs and prospective teachers was that prospective teachers possessed outcome expectancies that did not align with the goals of teacher education programs. They found that prospective teachers desired immediate answers to contextual situations that they had experienced rather than theory and subject content knowledge.

Research has shown that cognized goals are influential to motivation because explicit goals seen as challenging drive motivation (Bandura, 1993; Eccles & Wigfield, 2002). Bandura (1993) determined that challenging goals provide direction for behavior and create incentives for individuals to pursue and persevere until they attain their goals. In addition, he noted that challenging goals result in self-satisfaction that contributes to the level of one’s self-efficacy along with probable favorable outcome expectancies.

Harkness, D’Ambrosio, and Morrone (2007) discovered that prospective mathematics teachers who engaged in challenging mathematical tasks increased their mathematical self-efficacy. Struggling with and then accomplishing a challenging mathematical task provided them with the growing confidence to complete another challenging task. Eccles and Wigfield (2002) attributed motivation to challenging tasks, curiosity and interest, and the drive for mastery. Hiebert, Carpenter, Fennema, Fuson, Human, Murray, Olivier, and Wearne (1996) asserted that mathematics presented through problematic situations motivates prospective teachers and students, in their own learning situations respectively, to engage in and learn from their mathematical situations. Eccles and Wigfield (2002) declared that most individuals seek out stimulating and challenging activities because of a need for competence. Additionally, they asserted that when an individual feels competent and determined, she maintains her intrinsic motivation.

Schiefele (1999) discovered that interest correlated to conceptual learning more than surface learning. Schiefele asserted that individuals possess value-related valences to a task. Value-related valences refer to the feelings of value or significance an individual places on a task. Jansen (2008) attributed the value that an individual places on a task as a primary indicator of her interest in the task. She also linked utility value to motivation. She referred to utility value as the degree that the task helps an individual accomplish short or long-term goals. The higher utility value associated with a task, the higher degree of motivation one has in accomplishing the task.

Hiebert et al. (1996) acknowledged that a student's perception of a task is influenced by the values and expectations the teacher places on the task. Therefore, they declared that it is the teacher's role to assign a high utility value on mathematical tasks so that they influence students to hold the same value. In her study of the motivation of prospective teachers to participate in whole-class discussions, Jansen (2008) found that the prospective teachers who did participate did so because they had assigned a high utility value toward discussions. They attributed participating in discussions as a means to learning mathematics. Murphy (2006) discovered in her study of prospective teachers that they did not possess a high utility value toward learning mathematical content. They did not attribute learning mathematical content as a factor in their development as future elementary teachers. They placed a high utility value toward learning pedagogical knowledge. Mewborn (1999) noted similar findings with her prospective teachers. They were not inclined to want to learn about mathematics content and curriculum but were inclined to engage in learning about managerial issues.

Researchers have determined that a relationship exists between an individual's motivation and goals (Ames, 1992; Bandura, 1993, 1997; Dweck, 1986; Eccles & Wigfield, 2002; Schiefele, 1999). The type of goal an individual possesses determines his or her motivation toward completing a task. Ames (1992) distinguished between performance and mastery goals. He described performance goals as goals that individuals establish based on social contexts. Elliott and Dweck (1988) included the need to feel competent among peers or the decision not to participate for fear of appearing incompetent as performance goals. Mastery goals reflect goals that involve mastering tasks and understanding content (Ames, 1992; Harkness et al., 2007; Jansen, 2008).

Jansen (2008) discovered that the motivation for many prospective teachers to participate in whole-class discussions was to appear competent among their peers. Those who did not participate cited avoidance as a means not to appear less intelligent in front of peers. Mewborn (1999) found that prospective teachers were not inclined to engage in mathematical discourse with her and their collaborating teacher because they did not want to show mathematical weaknesses. She also discovered that prospective teachers' perceptions of authority influenced their use of avoidance strategies. Mewborn found that prospective teachers were unmotivated to participate in mathematical discussions because of their perceptions of the professor as an authority figure. They were concerned about her reaction to their responses. Harkness et al. (2007) found in their study of prospective mathematics teachers that having a mastery goal of conceptual learning of mathematical concepts motivated them to participate and complete tasks that were challenging and problematic. Furthermore, they discovered that prospective teachers who described themselves as possessing performance goals at the outset of the course described themselves as possessing mastery goals at the end of the course. Thus, their engagement in problematic mathematical situations altered their goals and increased their mathematical self-efficacy.

Mathematics for Teaching

The knowledge prospective teachers need to possess to become effective mathematics teachers has been debated in mathematics education literature (Borko, Eisenhart, Brown, Underhill, Jones, & Agard, 1992; Davis, 1997; Davis & Simmt, 2006; Hill et al., 2004; Hill et al., 2005; Hill et al., 2008; Korthagen & Kessels, 1999; Philipp, Ambrose, Lamb, Sowder, Schappelle, Sowder, Thanheiser, & Chauvot, 2007; Wilson & Ball, 1996). According to Hill et al. (2004), teacher knowledge has become such an object of concern that 38 states implemented assessments to measure teacher knowledge for certification. Research has suggested that an integrative approach of theory and practice could prevent the dominance of one type of teacher knowledge over another (Borko et al., 1992; Davis & Simmt, 2006; Hill et al., 2004; Korthagen & Kessels, 1999; Murphy, 2006; NCTM, 1991).

Mathematics education research has acknowledged the importance of subject content knowledge (Ball, 1990; Ball, 1991; Borko et al., 1992; Fennema, Carpenter, Franke, Levi, Jacobs & Empson, 1996; Hill et al., 2004; Hill et al., 2005; Hill et al., 2008; NCTM, 1991, 2009; Philipp et al., 2007; Simon, 1995; Shulman, 1986; Wilson & Ball, 1996). Fennema et al. (1996) cited teacher content knowledge as a major determinant of mathematics instruction and learning. NCTM (1991, 2000) listed content knowledge as an important element in effective mathematics teaching. Hill et al. (2005) were able to link teachers' content knowledge with achievement. In a study of 115 elementary schools, they found that teachers' content knowledge for teaching mathematics was a significant predictor of student gains. Furthermore, a teacher's mathematics preparation, the content and methods courses taken as a prospective teacher, also positively predicted student gains at the first grade and third grade levels. Hill et al. emphasized the significance of student gains at both grade levels by suggesting that teachers' content knowledge played a role even in the teaching of very elementary mathematics content.

Prospective teachers' mathematical content knowledge has been a concern in the mathematics reform effort (Ball, 1990; Ball, 1991; Borko et al., 1992; Fennema et al., 1996; Hill et al., 2005; Phillip et al., 2007; Simon, 1995; Shulman, 1986; Wilson & Ball, 1996). Philipp et al. (2007) found that prospective teachers' knowledge of conceptual mathematical understanding was limited. Ball (1990) found that prospective mathematics teachers' mathematical knowledge was rule-bound and compartmentalized. She discovered that most of the prospective teachers in her study could successfully solve a computation problem involving fractions, but few could explain the process or justify their solution. They relied on procedural knowledge to solve the problems, not conceptual knowledge. Tirosh (2000) arrived at a similar finding in her study of a prospective teacher's knowledge regarding division of fractions. Tirosh found that prospective teachers possessed adequate algorithmic knowledge but lacked intuitive and formal knowledge of mathematics.

Borko et al. (1992) discovered that prospective teachers' assumptions about teacher education limited the pedagogical content knowledge gained in the program. They found that prospective teachers' knowledge was procedural and that their pedagogical content knowledge was very narrow. Korthagen and Kessels (1999) warned about the danger of emphasizing procedural knowledge stating “student teachers learn a lot of methods and strategies for many types of situations but do not learn how to discover, in the specific situations occurring in everyday teaching, which methods and strategies to use” (p. 7).

Essential to a teacher’s understanding of mathematics is pedagogical content knowledge (Ball, 1991; Fennema et al., 1996; Graeber, 1999; Hill et al., 2008; Shulman, 1986). Graeber (1999) described pedagogical knowledge as the explanations, illustrations, examples and analogies a teacher relies on to make learning achievable for students. Davis and Simmt (2006) described this knowledge as requiring “knowledge of how mathematical topics are connected, how ideas anticipate others, what constitutes a valid argument, and so on” (p. 295). Hill et al. (2008) defined mathematical knowledge for teaching as “the mathematical knowledge that teachers use in the classroom to produce instruction and student growth” (p. 374).

Graeber (1999) listed several ideas that were essential to developing effective pedagogical content knowledge including a) understanding students’ current knowledge, b) distinguishing between skill and understanding, c) recognizing characteristics that promote retention, and d) providing alternate representations and recognition of alternative methods. Additionally, Borko et al. (1992) listed knowledge of representations and subject-specific knowledge of learners as critical components of pedagogical content knowledge.

Hill et al. (2008) identified knowledge of content and students (KCS) as an element within mathematics pedagogical knowledge. According to Hill et al., KCS is “content knowledge intertwined with knowledge of how students think about, know, or learn this particular content" (p. 375). Shulman (1986) defined this knowledge as being composed of "an understanding of what makes the learning of specific topics easy or difficult: the conceptions and preconceptions that student of different ages and backgrounds bring with them to the learning of those most frequently taught topics or lessons" (p. 9).

Mewborn (1999) found prospective teachers were concerned with understanding children's thinking when it differed from their own thinking. She discovered that prospective teachers found it difficult to explain a method for subtracting integers that differed from the one they had learned as a student. In their study of measuring pedagogical knowledge, Hill et al. (2008) discovered that most teachers possess the type of knowledge to represent certain topics using multiple representations. However, they noted that this type of knowledge development is not an explicit focus within mathematics coursework or mathematics teacher preparation courses.

Hill et al. (2004) created an assessment to gauge prospective teachers' mathematical knowledge for teaching. They surmised that common and specialized mathematical knowledge are related and intertwined but are not equal. Some teachers possessed strong content knowledge but lacked the specific type of knowledge necessary to teach. Furthermore, some teachers have developed specialized knowledge for teaching mathematics but lacked expert knowledge of mathematics.

Summary

The literature on mathematical discourse has become more extensive as the standards-based reform movement in mathematics teaching has progressed. Mathematics education research has demonstrated the benefits of students engaging in purposeful discourse toward their construction and understanding of mathematical concepts. Focusing on mathematical discourse toward construction of knowledge has been a relatively new concept for prospective teachers and teacher educators.

The literature has described challenges in fostering learning-oriented, mathematical discussions. A major challenge cited for teachers and teacher education programs is the move from low-level questioning to the development of questions that promote critical thinking. Research has suggested immersing prospective teachers in meaningful mathematical and pedagogical discourse. In doing so, prospective teachers could benefit in their own development of mathematical knowledge, and learn to recognize the characteristics and challenges of engaging students in active learning.

The types of teacher knowledge prospective teachers encounter in their preparation programs have been under more scrutiny since the movement toward reform-based mathematics. The debate has continued on whether mathematical content knowledge is more important than mathematical pedagogical knowledge toward teacher development. Advocates for the emphasis of content knowledge over pedagogical knowledge have argued that the lack of content knowledge makes the facilitation of student learning impossible. Proponents for mathematical pedagogical knowledge have claimed that mathematics content courses provide adequate content knowledge and teacher methods courses should focus exclusively on how to deliver content. Some researchers have argued that their integration provides prospective teachers with a perspective that places equal value on both types of knowledge (Korthagen & Kessels, 1999). One of this study's purposes is to evaluate the relationship of mathematical discourse on the acquisition of prospective teachers' development of teacher knowledge.

Theoretical Framework

Discourse within the Mathematics Classroom

According to NCTM (1991), "The nature of classroom discourse is a major influence on what students learn about mathematics" (p. 45). To understand the type of discourse occurring in the methods classroom, this study needed a framework that explained and categorized the patterns, functions, and characteristics of classroom communication. For this study, I used Brendefur and Frykholm's (2000) communication framework to categorize classroom discourse. Additionally, I relied on the work of Hatano and Inagaki (1991) to determine the hierarchy and flow of communication. Williams and Baxter's (1996) identification of instructional scaffolding within classroom discourse was used to identify the function of mathematical communication. Furthermore, I referred to Anderson et al.'s (2001) taxonomy of cognitive processes to identify the level of questions asked. Finally, Hill et al.'s (2008) mathematical knowledge for teaching provided a contextual understanding of the topics within the discourse.

Categories of Discourse

Brendefur and Frykholm (2000) developed a system containing four categories to identify the various levels of discourse. Each of the four categories builds upon the previous category. In other words, as one moves from one category to the next, the new category contains the characteristics of the previous category along with its new characteristics. Their categorization of classroom discourse began with the type of discourse that is characteristic of many teacher-student interactions. They described uni-directional communication as teacher-dominated because the teacher controls all levels of the discourse and students react to the questions and statements of the teacher rather than the statements given by their peers. Wertsch and Toma (1995) defined traditional classroom discourse as univocal, which means having one voice (as cited in Blanton et al., 2001). In traditional classroom discourse, the singular voice is that of the teacher.

In uni-directional discourse, the teacher is in full control of the nature and direction of the conversation. Brendefur and Frykholm (2000) discovered that students and teachers view the teacher as the authority of information. Lecture, closed questioning, and limited student opportunities for communication of ideas and strategies are common (Brendefur & Frykholm, 2000). Discourse is viewed a means to information received, encoded and stored (Blanton et al., 2001; Brendefur & Frykholm, 2000; Williams & Baxter, 1996). Brendefur and Frykholm (2000) noted that there are instances in which uni-directional communication is necessary. However, they have cautioned against its prevalent use. Though research has found uni-directional communication to be widespread, it has found that it limits opportunities for student construction of and contribution to mathematical learning (Cooney, Shealy, & Arvold, 1998; Thompson, 1984). Figure 1 illustrates the four levels of communication of discourse.

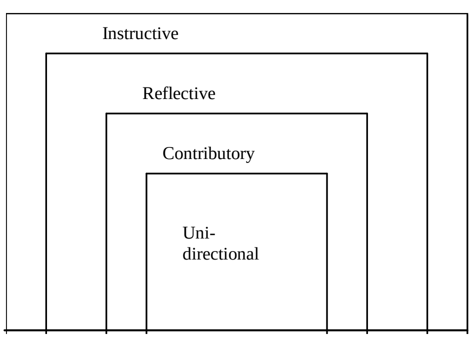

Figure 1. Brendefur & Frykholm's (2000) levels of communication.

Moving beyond uni-directional communication, discourse becomes contributive. Brendefur and Frykholm (2000) characterized contributive communication as "focusing on interactions among students and between teacher and students in which the conversation is limited to assistance or sharing, often with little or no deep thought" (p. 127). As the name has suggested, contributive communication moved the teacher from completely dominating the discourse to affording students a role in contributing to the conversation. The view of the teacher continues to be the authority of information. In an environment of contributive communication, students have opportunities to discuss, help, and work together on tasks. While the teacher does not completely dominate the interactions, she still provides direction and limitations to student interactions. The teacher continues to play a dominant role in controlling the nature of the discourse. Conversations are mostly corrective or explanatory. Brendefur and Frykholm have noted that conversations within contributive communication lack the depth and intensity of connecting tasks or problems to mathematical concepts or ideas.

Unlike uni-directional and contributory communication, Brendefur and Frykholm have stated that a conceptual shift takes place as discourse moves into the next two categories. Reflective communication allows mathematical conversations to go beyond procedural information to deeper investigations. It contains the same attributes of contributive communication, but allows solutions or procedures to become a jumping-off point for discussions on ideas and concepts. Cobb et al. (1997) described this level of communication as "repeated shifts occur such that what the students and teacher do in action subsequently becomes an explicit object of discussion" (p. 258). Morrone et al. (2004) described this type of discourse as "instances where the teacher presses a student to elaborate on an idea, attempts to encourage students to make their reasoning explicit, or follows up on a student's answer or question with encouragement to think more deeply" (p. 29).

Brendefur and Frykholm (2000) characterized the environment in which reflective communication exists. They stated, “Students are asked to not only share information but to think about what was said, to incorporate those ideas into their own, and to build upon the conversation in meaningful ways” (p. 149). Lampert (1990) identified reflective communication within the context of students justifying or arguing about solutions and conjectures presented by their peers. Wertsch and Toma (1995) described reflective discourse as dialogic wherein “text becomes a starting off point for making sense of an idea or constructing new ideas” (as cited in Blanton et al., 2001, p. 230).

Simon (1995) has pointed out that while reflective communication might exist in a classroom, it might not exist for all students. Cobb et al. (1997) described the relationship between reflective discourse and conceptual development as “speculative” (p. 265). In other words, some students might reach a level of reflection from the conversations that occur in the classroom while others may not. This suggests that reflective communication may support student reflection and understanding of mathematical concepts, but it does not determine it. According to Cobb et al., “It is the individual child who has to do the reflecting and reorganizing while participating in and contributing to the development of the discourse” (p. 266).

Brendefur and Frykholm (2000) identified the final level of classroom discourse as instructive communication. It is the apex of classroom discourse. As the name has implied, instructive conversations either cements mathematical thinking or modifies thinking of mathematical ideas. Nathan and Knuth (2003) described instructive communication as “forms of social exchange which provides participants with an avenue to construct and build on correct conceptions through their interactions” (p. 204).

Steffe and D'Ambrosio (1995) described the power of instructive communication as it relates to student learning. Through instructive discourse, conversations "serve as instruments of communication and as a means for the teacher to support and sustain the students' mathematical task" (p. 158). Morrone et al. (2004) labeled this type of discourse as "higher order thinking…instances when the students are asked to display deeper understanding, to make connections of the current topic to their prior knowledge, or to think about relationships between ideas" (p. 29).

In an environment where discourse has reached the instructive level, the teacher has allowed the conversation to dictate the path of the classroom. Mathematical conversations have the potential to steer away from the topic of instruction or be unplanned. Morrone et al. noted that based on the conversation, instructive communication should result in deeper understanding of mathematical ideas and concepts that is constructed by students. Brendefur and Frykholm (2000) declared that the goal of instructive communication is to alter the experience of the classroom because of the conversations within the classroom.

Discourse Hierarchy

In their work on discourse, Hatano and Inagaki (1991) have identified the existence of hierarchical levels of conversations. They found that information within a classroom flows in two directions, either vertically or horizontally. Vertical discourse occurs when information flows between a person considered more knowledgeable to a person of less knowledge. Within the classroom context, vertical discourse traditionally has occurred between the teacher and student. Researchers have found that vertical discourse is more prominent in environments where uni-directional and contributory communication exist (Brendefur & Frykholm, 2000; Cobb et al., 1997; Wood, Cobb, & Yackel, 1993). Hatano and Inagaki (1991) have declared that vertical discourse is necessary in some situations and even has the opportunity to be productive.

Social constructivism research has placed more emphasis on the horizontal discourse that occurs in the classroom (Hatano & Inagaki, 1991; Simon, 1995; Wood et al., 1993). Hatano and Inagaki (1991) have defined horizontal discourse as conversations that occur between peers who are at a comparable level of expertise. In the classroom context, horizontal discourse occurs among students. NCTM (1991) called for teachers to allow more opportunities for student-to-student discourse. However, teachers have struggled with moving away from vertical discourse as the dominant form of communication. In a study conducted by Nathan and Knuth (2003), over a two-year period, a teacher increased the horizontal discourse in her classroom from 1% to 33%. While this signified a marked increase, it still demonstrated the dominance of vertical discourse in classroom communication.

The Nature of Discourse

Regardless of the type of discourse or the instigator of communication, discourse contains a purpose. Williams and Baxter (1996) discovered that in many instances, mathematical conversations have been found to contain scaffolding of conceptual and procedural information. Patrick (1997) defined scaffolding to include “any evidence of the teacher providing additional support for learning by modeling, outlining, use of questioning procedures, or suggesting where to find additional help” (as cited in Morrone et al., 2004, p. 29). Williams and Baxter have noted the benefits for students in their knowledge construction when the teacher appropriately provides scaffolding.

Williams and Baxter (1996) discussed two types of instructional scaffolding-analytical and social. They defined analytical scaffolding as “the scaffolding of mathematical ideas for students” (p. 24). Nathan and Knuth (2003) cited a teacher restating a student statement within a discussion for clarity or incorporating mathematical terminology as examples of analytical scaffolding in the mathematics classroom. NCTM (1991) asserted that an important aspect in promoting mathematical discourse is to ensure mathematical clarity and understanding of concepts. Williams and Baxter (1996) supported the use of analytical scaffolding. They explained that “such scaffolds allow students a framework on which they can begin to build their own knowledge and provide help in organizing their thinking, with the goal of gradually removing the scaffolding and allowing full ownership of the constructed knowledge” (p. 25). Scaffolding represents a ladder of knowledge that eventually is removed by the teacher as students begin constructing their knowledge.

Williams and Baxter (1996) defined social scaffolding as “the scaffolding of norms for social behavior and expectations regarding discourse” (p. 24). Nathan and Knuth (2003) included a teacher asking students to explain their solution or asking for class participation in a discussion as examples of social scaffolding. Williams and Baxter (1996) cited the establishment of valid mathematical discourse or the setting of class expectations as examples of social scaffolding. In their definition of discourse-oriented teaching,. Hufferd-Ackles et al. (2004) linked student growth in explaining their mathematical ideas to “the social climate that the class together developed that effectively supported student explainers” (p. 97). NCTM (1991) supported the use of social scaffolding to create an environment of respect and civility.

The balance in incorporating both forms of scaffolding effectively and efficiently has been a challenge for teachers. Researchers have documented the struggle teachers face with instructional scaffolding (Nathan & Knuth, 2003, Williams & Baxter, 1996). Nathan & Knuth cited a teacher's struggle with scaffolding. The teacher aimed at acknowledging all students' contributions to a discussion and yet, did not acknowledge the mathematics that was worthy to the discussion. Her focus on instructional scaffolding, establishing the climate and expectations for student interaction, prevented the teacher from focusing on the mathematics of the discussion. Likewise, on the other end of the continuum, some teachers have aimed to put learning completely in the hands of students and have distanced themselves from applying any type of analytical scaffolding (Williams & Baxter, 1996). Williams and Baxter noted that establishing norms of discourse have presented difficulties due to the accustomed nature of mathematical classrooms where uni-directional, vertical discourse has been overwhelmingly present.

Questioning

Bloom, Englehart, Furst, Hill, and Krathwohl (1956) created a taxonomy of educational objectives that has classified educational questioning for the last 50 years. Bloom's taxonomy differentiates between multiple levels of thinking and knowledge ranging from levels that required little intellectual processing to levels that integrated multiple processes, knowledge, and skills. The relationship between questioning and Bloom's levels of cognitive domains existed in the assumption that the cognitive level of the question determined the response. Bloom et al. combined defining questions with their responses within the levels because responses are an intended behavior of the question. A response that is factual in nature, and categorized at a knowledge domain, would have likely resulted from a question within the same domain. Likewise, a question that required an evaluation of an idea would yield an evaluative response, more so than a question that asked for understanding of a concept.

Anderson et al. (2001) have presented a revision of Bloom's taxonomy. A main difference between Bloom et al.'s (1956) original taxonomy and the revised taxonomy is the separation between the noun and verb of each dimension level. The revised taxonomy has two dimensions-knowledge and cognitive process. For every educational objective, the noun represents what an individual should know about a topic (knowledge) and the verb (cognitive process) represents what the individual should be able to do with that knowledge (Krathwohl, 2002). Their revised taxonomy contains four levels within the knowledge dimension and six levels within the cognitive process dimensions.

Within the knowledge dimension, Anderson et al. (2001) identified four categories. The first category they identified is *factual knowledge.* Factual knowledge represents the basic elements that individuals must know to be acquainted with the content. They described factual knowledge as the information that students memorize such as multiplication facts. The second category they identified is *conceptual knowledge*. Conceptual knowledge represents the interrelationships among the basic elements within a larger structure. Conceptual knowledge includes classifications, categorizations, and theories. This knowledge links details and facts to a larger idea.

The third category Anderson et al. (2001) identified is *procedural knowledge*, which is the knowledge of algorithms and methods. It is the knowledge of how to do something. Anderson et al. identified the last category in the knowledge dimension as *metacognitive knowledge*. Metacognitive represents the highest level of knowledge an individual can possess. They defined metacognitive knowledge as the knowledge of cognition as well as the knowledge of one's own cognition. In other words, being aware of one's own learning allows one to adapt her or his ways of thinking.

The cognitive process dimensions categorize the processes that learners perform with the information they encounter during instruction. Anderson et al. (2001) described the cognitive processes as the verbs of the educational objectives that signify what the learner does with her or his knowledge. For one to obtain a higher level of knowledge, higher cognitive processes must have occurred.

At the lowest level of the cognitive process dimension, Krathwohl (2002) defined *remember* as "retrieving relevant knowledge from long term memory" (p. 215). This cognitive process includes the recall of facts and information. Moving up the cognitive ladder is the process, *understand*. The verb, *understand*, has the potential to carry multiple meanings ranging from interpreting and classifying to summarizing and comparing. Krathwohl described the overall process, *understand*, to "determine the meaning of instructional messages" (p. 215). Anderson et al. (2001), described the process as complex because some of the processes within this domain are more complex than the processes in the next domain depending on their use.

Moving from *remember* and *understand* to the following processes signifies a shift in critical thinking. Krathwohl (2002) noted that the next processes require individuals to form meaning and use that meaning in the context of other situations. Anderson et al. (2001) defined *apply* as the process of carrying out procedures or algorithms. Therefore, at the *apply* level, an individual has to remember an algorithm, understand its meaning, and use the algorithm in the context of a problem in order to reach a solution. Anderson et al.'s revised taxonomy described *analyze* as breaking material into its parts and determining the relationship of the parts. Krathwohl suggested that this process often involves inductive and deductive reasoning.

Anderson et al. (2001) identified the highest levels of the cognitive process as *evaluate* and *create*, respectively. They asserted that the process, *evaluate*, requires individuals to make a judgment based on the previous knowledge and process domains they have encountered. They described the process, *create*, as the combination of all the domains that together form a new, original product. The process, *create*, represents a true, deep conceptual understanding and application of knowledge. Krathwohl (2002) noted that this highest-level of thinking is not accomplished successfully without the progression up the ladder of the prior two dimensions.

Forms of Teacher Knowledge

As Shulman (1986) analyzed research on teacher education, he found that the literature failed to address a teacher's knowledge on her chosen subject matter. Before his work on teacher knowledge, the type of knowledge teachers needed to be effective in the classroom had not been explicitly described (Hill et al., 2008). Shulman lamented, "What we miss are questions about the content of the lessons taught, the questions asked, and the explanations offered" (p. 8). He questioned the transition from expert student to novice teacher. He surmised that the identification of this knowledge could transform the manner in which prospective teachers prepared for the classroom. Through his analysis of teacher knowledge, he identified three core categories: a) subject matter content knowledge, b) pedagogical content knowledge, and c) curricular knowledge. Shulman suggested that an effective teacher must possess all three types of teacher knowledge.

Hill et al. (2008) expanded upon Shulman's definition of teacher knowledge adding an additional three dimensions that provided further depth and insight into the nature of mathematical teacher knowledge. Ball (1990) likened mathematical content knowledge as knowledge *of* mathematics and mathematical pedagogical content knowledge as knowledge *about* mathematics. Hill et al.'s three additional dimensions include: a) knowledge about content and students at different developmental stages, b) common content knowledge among teachers of mathematics, and c) knowledge at the mathematical horizon. The components of subject matter and pedagogical content knowledge provide a comprehensive view of mathematical teacher knowledge. Figure 2 reflects the mathematical knowledge for teaching domains created by Hill et al. (2008).

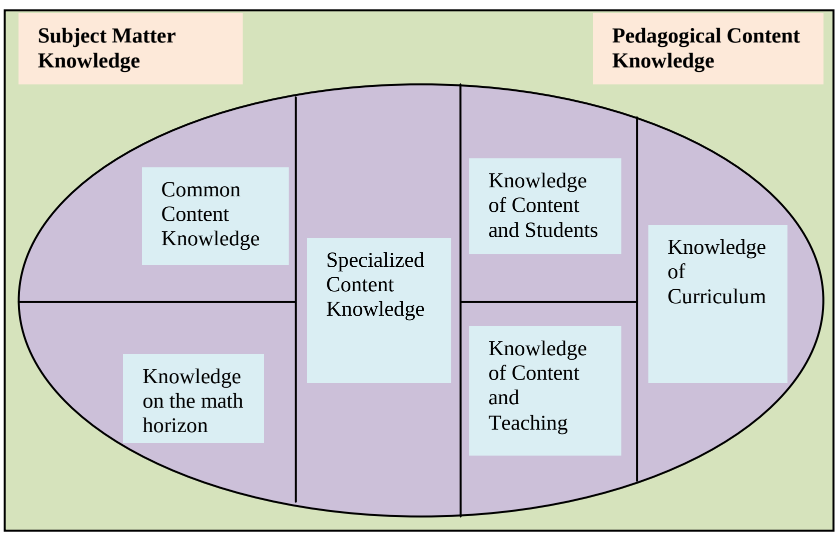

Figure 2. Domain map for mathematical knowledge for teaching created by Hill et al (2008).

Hill et al. (2008) described subject content knowledge as not only knowledge of the subject but also the special forms of mathematical knowledge specific to teaching. For the mathematics teacher, the recognition of various ways of viewing mathematics–as a body of knowledge or as an avenue for critical thinking is important. A mathematics teacher has to be prepared to explain the rules of subtracting integers and explain why other competing explanations of subtracting integers are invalid.

Hill et al., working from Shulman's conception of content knowledge, identified two specific types of content knowledge particular to teaching. They identified common content knowledge (CCK) as knowledge used to teach mathematics that is common to anyone who is in a mathematics-related profession. This mathematics unites mathematics educators with mathematicians. They identified specialized content knowledge (SCK) as the mathematical knowledge that allows teachers to engage in particular teaching tasks that include providing multiple representations and explaining why implementing a procedure or algorithm works toward a solution.

Content knowledge encompasses the *what* and the *why*. Likewise, content knowledge is more than facts or concepts. Hill et al. (2005) renamed content knowledge as mathematical knowledge for teaching, defining it as "the mathematical knowledge used to carry out the work of teaching mathematics" (p. 373). This mathematical knowledge includes "explaining terms and concepts to students, interpreting students' statements and solutions, judging and correcting textbook treatments of particular topics, using representations accurately in the classroom, and providing students with examples of mathematical concepts, algorithms or proofs" (Hill et al., 2008, 373). Content knowledge is possessing the conceptual understanding for rules and algorithms. For teachers to be effective, they have to deliver and teach concepts, explain why concepts are true and valid within a particular mathematical system, why others perspectives within a concept are invalid, and signify the importance of the concept.

Pedagogical Content Knowledge

Researchers have found that pedagogical content knowledge differs from content knowledge because pedagogical content knowledge focuses on content knowledge specifically for teaching (Graeber, 1999; Hill et al., 2008; Shulman, 1986; Simon, 1995). Graeber (1999) described pedagogical content knowledge as the various ways mathematics teachers make mathematics learning achievable for students. Hill et al. (2008) defined the concept of pedagogical content knowledge to include a body of knowledge that intertwines content with knowledge about student learning within a specific content. They described knowledge of content and students (KCS) as "being used in tasks of teaching that involve attending to both the specific content and something particular about learners…" (Hill et al., 2008, p. 375). They noted that this form of pedagogical knowledge is applied when thinking about common misconceptions or applications that prevent students from internalizing the concept correctly. When dividing, students often think that the quotient is always smaller or when adding or subtracting fractions, students perform computations to the denominators. Possessing this knowledge, separate from content knowledge or general pedagogical knowledge, would allow a teacher to plan tasks accordingly to address and rectify misconceptions.

Graeber (1999) noted that pedagogical content knowledge includes knowledge of why particular concepts are difficult for students to master. Ball (1991) stated that it encompasses recognizing and addressing the assumptions and misconceptions students might have that inhibited their acquisition of the concept. Research has identified addressing misconceptions as an essential component to the call for strong pedagogical content knowledge (Graeber, 1999; Hill et al., 2008; Shulman, 1986).

Curricular Knowledge

Shulman (1986) identified curricular knowledge as the third essential category of teacher knowledge. Hill et al. (2008) included curricular knowledge as one of their categories of mathematics knowledge for teaching. They defined curriculum knowledge as “awareness of how topics are arranged both within a school year and over time and ways of using curriculum resources, such as textbooks, to organize a program of study for students” (p. 377). Shulman described the possession of curricular knowledge to include knowledge about the content curriculum and the materials used to deliver the curriculum to students. Nicol and Crespo (2006) defined curriculum as the set of topics and lessons embedded within each grade level standard. They noted that each state has adopted a set of curriculum standards. For example, the Georgia Department of Education (2010) adopted the Georgia Performance Standards (GPS) that are standards that indicate the level of mastery of academic content for each grade level. Student mastery of the GPS standards has been determined by a standardized test that is administered each year. The Georgia Department of Education (2010) has adopted the Common Core Standards (CCS) that more than half of the 50 states will implement. This set of standards reflect concepts and skills that students nationwide will learn for mastery.

Along with the curriculum, Nicol and Crespo (2006) noted that teachers receive various instructional materials that aid in their implementation of the curriculum. Such materials include textbooks with aligned teacher resources. They also included available software programs, websites, tutorials, graphing calculators and other instructional tools as instructional resources. In their research, Nicol and Crespo warned of the increased role textbooks have in promoting teacher learning and guided instruction. As they discovered, the textbook especially took on the role of the curriculum guide for teachers. Therefore, they suggested that teacher educators include being able to adapt and elaborate the textbook and other curriculum materials based on pedagogical knowledge as focus for teacher development. Shulman (1986) likened the curriculum and associated materials as "the pharmacopoeia from which the teacher draws those tools of teaching that present or exemplify particular content and remediate and evaluate the adequacy of accomplishments" (p. 10). Curriculum materials have the potential to enhance instruction, but cannot replace instruction embedded with strong content knowledge and effective pedagogical skills.

CHAPTER III

METHODOLOGY

Restatement of Research Questions

The purpose of this study was to analyze the classroom discourse of a mathematics methods course. This study centered on investigating and answering the following questions regarding the discourse in a graduate mathematics methods course:

1. How does discourse appear within the instructional environment of a mathematics methods course for prospective teachers?
    a. What are the patterns, functions, and nature of classroom discourse?
    b. What role does questioning play in shaping classroom discourse?
2. How did the discourse in the mathematics methods course facilitate or hinder opportunities for prospective teachers to develop mathematical knowledge for teaching?

In this chapter, I outline the methodology I used to conduct the research related to this study. Specifically, this chapter discusses in detail the following components: a) my research paradigm including my philosophical assumption that guided the study, b) the selection of a case study as my research design, c) the participants of the study including their description, d) the setting of the study, and e) the instrumentation and methods used to collect data. The chapter concludes with a detailed description of the analysis of the data and measures to ensure trustworthiness.

Research Paradigm

For this study, a qualitative research approach was used to analyze the discourse within a mathematics methods course. A qualitative approach was selected because my purpose was to contribute to the understanding of a phenomenon (Creswell, 2007). This understanding requires the deep exploration of the thoughts and ideas of the participants involved. Marshall and Rossman (1995) described qualitative research as “exploratory or descriptive and that stresses the importance of context, setting, and the participants’ frame of reference” (p. 44). These important qualities could not have been achieved as effectively through quantitative measures alone. Yin (2003) noted that pure quantitative measures do not provide the depth of the context nor do they allow for the individuality of the participants. For this study, I observed participants in their setting, analyzed written documents such as the class syllabus and mathematical tasks, gathered prospective teachers’ thoughts on their discourse through written prompts, interview responses from three prospective teachers, and audio-recorded conversations that occurred during the mathematics methods class.

When conducting qualitative research, a philosophical assumption lays the foundation for the study (Creswell, 2007). In this study, an ontological assumption was the underlying assumption. An ontological assumption focuses on the multiple realities of the participants (Creswell, 2007; Marshall & Rossman, 1995; Taylor & Bogdan, 1984). An ontological assumption uses quotes and themes generated from the participants as evidence. It values the time and space in which the participants lived. This study took the form of interpretive research. Interpretive research assumes that reality was socially constructed and relied on multiple realities (Merriam, 2009). Interpretive research, as conducted in the this study, seeks to understand experiences. Creswell (2007) explained interpretive research by stating, “In this worldview, individuals seek understanding of the world in which they live and work. They develop subjective meanings of their experiences” (p. 20). According to Guba (1990), a paradigm is a basic set of beliefs that guide action. In a pragmatist paradigm, the overlying concern is on the outcome of the research (Creswell, 2007). A pragmatist paradigm guided this study. The importance was on how the data and themes had the potential for future implications in research and practical modifications in the field. The purpose of this study was to evaluate the discourse within a mathematics methods course and the type of mathematical knowledge for teaching the discourse facilitated. In holding a pragmatic perspective, a strong focus was the practical solutions and answers to the research questions the data provided. The practical outcome available through this study included the possible modification of the content of future teacher education preparation courses and an increased emphasis on teaching discourse skills through the facilitation of purposeful discourse to serve as a model for prospective teachers.

Research Design

According to Yin (2003), a research design provides the link between the data collected and the questions asked. It provides a roadmap on how to get from point A to point Z. A research design also acts as a blueprint for the study. A purpose of a research design is also to demonstrate that the researcher has the ability to conduct qualitative research (Marshall & Rossman, 1995). Marshall and Rossman have categorized this as the “do-ability” of the qualitative research study (1995, p. 6). The research design for this study was a single case study. Creswell (2007) has defined a case study as “a qualitative approach in which the investigator explores a bounded system (a case)…over time, through detailed, in-depth data collection involving multiple sources of information, and reports a case description and case-based themes" (p. 73). Bogdan and Biklen (1992) compared a case study to a funnel describing it as having a wide opening and ending with a narrow focus. According to Yin (2003), "As a research strategy, the case study is used in many situations to contribute to our knowledge of individual, group, organizational, social, political and related phenomena" (p. 1). Merriam (2009) defined a case study as "an in-depth description and analysis of a bounded system" (p. 7). Yin (2003) suggested the use of a case study when "a 'how' or 'why' question is being asked about a contemporary set of events in which the investigator has little or no control" (p. 9). Marshall and Rossman (1995) suggested that the use of a case study is also appropriate when an issue warrants deep exploration.

This study was a single instrumental case study because it examined the issue of classroom discourse within one particular mathematics methods course over the course of three class meetings. In particular, this study was an observational single case study. Bogdan and Biklen (1992) defined observational case studies as those in which "the major data gathering technique is participant observation supplemented with formal and informal interviews and review of documents, and the focus of the study is on a particular organization or some aspect of the organization" (p. 60).

A characteristic of this study included its description as heuristic. According to Merriam (2009), "Heuristic means that case studies illuminate the reader's understanding of the phenomenon under study. They can bring about the discovery of new meaning, extend the reader's experience, or confirm what is known" (p. 44). Specifically, this study allowed for the examination of the nature and function of the classroom discourse, how prospective teachers engaged in mathematical discourse within the class and the level of significance the discourse had toward the purpose and goals of the class.

Within a case study, the blueprint of the research design consists of propositions, units of analysis, and theory development (Yin, 2003). Merriam (2009) characterized the unit of analysis, the case, as the single most defining characteristic of a case study. She also noted the criteria of a unit of analysis stating, "For it to be a case study, *one* particular program or *one* particular classroom of learners, or *one* particular older learner selected on the basis of typicality, uniqueness or success would be a unit of analysis" (p. 41).

The study's unit of analysis was a single unit of analysis meaning that the study contained only one focus. In this study, the unit of analysis consisted of the discourse of the prospective teachers and their professor in the mathematics methods course. Data were collected over the course of three weeks. I observed the mathematics methods class three times for three Mondays for two hours and forty-five minutes each time. During these visits, I audio-recorded class conversations, gathered artifacts from the class such as copies of tasks, and took field notes of my observations. After each observed class, the prospective teachers completed a writing prompt reflecting on their participation in the discourse of the class meeting.

Yin (2003) has noted that theory development affirms the significance of theory within the study. Additionally, it affirms how the study contributed to research in new ways (Marshall & Rossman, 1995). Theory development is the formation of theory or the expansion of theory through well-conceptualized and well-conducted research. Yin

(2003) asserted that theory development allows for generalizations of the data collected and grounds the study. The theories that underpinned this study included Brendefur and Frykholm's (2000) categorization of communication, Hatano and Inagaki's (1991) hierarchy of discourse, Williams and Baxter's (1996) framework of instructional discourse, Anderson et al.'s (2001) revision of Bloom's taxonomy, along with Hill et al.'s (2008) components of teacher knowledge. Each of these theories were discussed in detail in Chapter Two of this study.

The Setting

The setting provides the reader with a picture of the environment under which the study took place (Bogdan & Biklen, 1992). Taylor and Bogdan (1984) characterized the ideal setting for a qualitative study as one "in which the observer obtains easy access, establishes immediate rapport with informants, and gathers data directly related to the research interests" (p. 19). This study met each of these criteria.

Easy access was obtained through the agreement of participation from the professor and subsequently, the prospective teachers. I initially emailed the professor of the course my purpose and intentions regarding data collection. Upon her agreement, I attended the class and described my purpose and intent to the prospective teachers. They agreed to participate in the study. Upon their agreement, they all signed consent forms. The participants were aware that their conversations and actions would be confidential.

Through my personal connection with two of the prospective teachers, I was able to establish a rapport with the rest of the participants. One of the prospective teachers was completing her field experience for the methods class at the school in which I was an administrator. Therefore, I became acquainted with her from her attendance at various meetings at my school. Another prospective teacher worked at the same school as my husband. Through these connections, I was able to weave into conversations with the prospective teachers and spark relationships.

The last criterion for easy access was the gathering of data directly related to the research interests. By physically observing the classes, I was able to gather field notes and record the discourse. This allowed me to actively see and hear the prospective teachers in action. Doing so provided me with data that directly related to my study.

A large university in a suburban area of the southeastern United States was the regional backdrop for this study. At the time of this study, the university held an enrollment of 19,844 students. The website, StateUniversity.com (2010) described the university as a large four-year, primarily non-residential university. Education was a popular major with elementary education ranking as the second-highest most popular field of study. The university held the second largest education program in its state.

This large southeastern university created the MAT program to provide initial teacher certification to college graduates who held bachelor degrees. The MAT program offered certification in six contents areas, including mathematics. The courses within the MAT program were graduate courses. MAT candidates received their master’s degree upon their completion of the program. Mathematics was chosen as the subject of study because of my previous experience as a mathematics teacher. I also decided to focus on mathematics because my major concentration in my doctoral program was mathematics. The methods course within the mathematics education program was selected as the setting due to my interest in the fusion of mathematical content with pedagogy.

The mathematics methods course met weekly on Monday nights. Each class lasted close to three hours. The classroom environment was casual. Since the class size was small, the class met in a conference room. The conference room consisted of a large round table, a whiteboard, and a LCD projector. The prospective teachers sat in the same location each class. They often engaged in conversation before class officially began with each other and with the professor. Some of these conversations related to the content of the class while others were not. The prospective teachers possessed a class syllabus that listed the tasks and agenda for each class meeting. Assignments, discussions about readings, and instructional tasks comprised the agenda for each session. These tasks were both mathematical and pedagogical in nature. Since different school systems incorporated various curricular materials, the course explored curricular issues broader in nature such as state standards and the national Common Core mathematics standards.

The Participants

Sampling

The prospective teachers in this study were enrolled in their initial mathematics methods course. This mathematics methods course served as the bounded system, or case, for this study (Bogdan & Biklen, 1992; Merriam, 2009). The process of sampling involved choosing what, where, when, and whom to observe (Merriam, 2009). The sampling process that took place in this study was a combination of purposeful and convenience sampling. The selection of the course and professor was purposeful. However, I selected the prospective teachers in the class as participants through convenience sampling.

Merriam (2009) defined purposeful sampling as “the assumption that the investigator wants to discover, understand, and gain insight and therefore must select a sample from which the most can be learned” (p. 77). One type of purposeful sampling is the unique sample. A unique sample is based on unique characteristics of the phenomenon studied. While classroom discourse, and more specifically, mathematical discourse was an important phenomenon in this study, the sample of the mathematics methods course was a unique sample in regard to all mathematics graduate courses and to all initial-certification-seeking mathematics students. I utilized convenience sampling to select the participants of the study, excluding the professor. All of the prospective teachers who enrolled for the mathematics methods course were selected as participants because the sample included everyone who enrolled for the course.

Once I selected the participants, the prospective teachers and the professor of MAT mathematics methods course, further sampling was completed to determine when to complete the participant observations. Before the study began, I received IRB approval to begin collecting data at the mid-point of the semester. Therefore, there was only an opportunity to observe six class meetings. Thus, I selected the class meetings I could observe through convenience sampling. I coordinated the available dates left in the semester, and the number of times that I would observe with my availability to be present at the class meetings to determine which classes I observed.

Participant descriptions

The participants were comprised of six female, one male prospective teacher, and one female professor. Five of the seven prospective teachers were in their 20’s, one was in her 40’s, and one was in his 60’s. Lisa was a Caucasian prospective teacher in her 20’s.

She always sat near the professor at a circular table in the conference room they used for class meetings. Lisa held a degree in mathematics and was at the time, completing her field experience in a middle school setting teaching gifted students. Lisa was open and involved in the class environment. Jill was a Caucasian preservice teacher in her 20's. She sat next to Lisa and was also in a middle school setting teaching sixth grade mathematics. Jill's degree was in mathematics. Jill was energetic and asked the professor questions when she questioned an idea or sought guidance on an issue. Amy was an African-American preservice teacher in her 20's. Amy already held a provisional teaching certificate and was teaching in a local high school. Amy was vocal and dominant in the class's discussions. She commented frequently during class. Amy was a focal point within the class environment.

Ann was a Caucasian prospective teacher in her 40's and a mother of two teenage children. She had a degree in statistics. Ann sat next to Jill but frequently talked to Amy as they shared some commonalities such as Amy being a teacher at the school Ann's children attended. Ann was completing her field experience in a seventh grade mathematics classroom and in particular, the middle school where I was an administrator. Mike was the sole male prospective teacher. He was African-American in his late 60's. Mike possessed a Masters in Business Administration degree. He was reserved and spoke minimally during class. Mike was completing his field experience in a high school setting. Beth was a Caucasian prospective teacher in her 20's. Beth held a degree in mathematics and was in a local high school for her field experience. She spoke infrequently during class and in conversations with her peers during breaks. Maddie was a Caucasian prospective teacher in her 20's. Maddie sat next to the professor and always spoke directly to the professor. She was situated in a high school setting for field experience teaching Math I, a freshman integrated mathematics course. She was not vocal during discussions but would comment on topics or others' statements.

Finally, Dr. Peters was an energetic, intelligent, and kind African-American professor in her 40’s. She held a doctoral degree in mathematics education. Her previous teaching experience at the university included college algebra and trigonometry, a mathematics for teaching course, as well as a series of three courses in mathematics for elementary teachers. This was her first experience teaching the MAT mathematics methods course although she had vast experience teaching mathematics methods courses at the undergraduate level.

Instrumentation

Classroom Observation Protocol

A focus of the research was to analyze the characteristics of the discourse within a mathematics methods course. Therefore, I needed an organizational tool that would allow me to record field notes of what I saw and heard while being a participant observer. Merriam (2009) described field notes as the raw data from which findings emerge. I created a discourse observation form based on an observation chart created by Artzt and Armour-Thomas (2002). The observation chart included in NCTM’s (2008) *Purposeful Discourse* was appealing in its simplicity.

The observation chart I created (See Appendix C) contained five categories that structured, but did not limit, the observations. The first column provided a description of the observed task. This allowed me to document the specific content or topic associated with the task, any performance or process standards the task addressed, as well as the directions for the task. The second column categorized the type of discourse as either *vertical* or *horizontal* as defined by Hatano and Inagaki (1991). Based on their definition of vertical discourse, I viewed vertical discourse as discourse that occurred between the professor and the prospective teachers. Based on their definition of horizontal discourse, I viewed horizontal discourse as discourse that occurred between the prospective teachers.

The third column categorized the type of teacher knowledge associated with the task. Did the task focus on a mathematical concept, curricular issue, or a pedagogical topic? This categorized the task as either *content*, *pedagogical*, or *curricular*. This categorization aligned with the framework for teacher knowledge described by Hill et al. (2008). Content knowledge, as defined by Shulman (1986), is knowledge that furthers the development of understanding about a specific content. In the context of this study, content knowledge was mathematical knowledge; knowing how to do mathematics. Shulman defined pedagogical knowledge as knowledge about how to teach a specific content. He defined curricular knowledge as knowledge about the content curriculum and the materials used to deliver instruction of the content.

The fourth column to characterize the participants' level of communication during the task. The participants' level of communication was categorized as *uni-directional*, *contributory*, *reflective* or *instructive*. These levels of communication were established by Brendefur and Frykholm (2000) in the theoretical framework section of Chapter Two. Each level is successive starting at uni-directional, which is professor-centered to instructive, in which the prospective teachers are constructors of their own learning and knowledge.

The final column of the observation chart provided space to note any descriptors, questions, thoughts, or observations that did not fall under one of the columns. I also noted key questions asked during the discourse. I categorized key questions by their cognitive level based on Anderson et al.'s. (2001) taxonomy. Their revised taxonomy measured the cognitive level of questions based on the process the verb of the question demanded. I assigned questions a low cognitive level if they asked for recall or regurgitation of information. Anderson et al. classified these questions as *remember* or *understand* cognitive processes. Conversely, questions were determined to be at a high cognitive level if they required the prospective teachers to make evaluations, or use knowledge to create a new product. They classified these types of questions as *apply, analyze, evaluate* or *create* cognitive processes.

Interview Protocol

A second piece of instrumentation utilized to gather data was an interview protocol. The interview protocol (See Appendix D) was designed to capture the perspective and thoughts of the prospective teachers. I developed the interview protocol after the initial data collection and initial findings. The initial findings prompted further questions. I created each question based on an issue relevant to the classroom discourse in a mathematics methods course.

The interview protocol contained six primary questions. The first question asked the three prospective teachers to state what they considered the purpose of the course. I wanted to know their expectations of the course. The second question asked them to state the information they had gained from the course that they believed has helped them to develop as future educators. The third question asked the three prospective teachers to describe the level of importance they placed on mathematical discourse in the classroom. The fourth question asked the three prospective teachers to state and describe the knowledge they considered essential knowledge for their development as future educators. The fifth question asked them to rate their own confidence in terms of their mathematical ability. The last question asked the prospective teachers to make a conjecture as to why there was little observed discourse about the mathematical tasks they completed in the class.

The responses to these primary questions led to four follow-up questions for the participants. The first question asked them to state whether they believed the focus of the class revolved around the development of knowledge of mathematical content, mathematics pedagogy, or operational issues such as classroom management. The second follow-up question asked the participants to state any information or skills that they had hoped to gain from the methods class, but did not.

The next two follow-up questions revolved around their reported mathematical confidence. One of the purposes of the interviews was to gain insight into the reasons for the lack of mathematical discourse. I asked the participants to state whether they believed they had a strong conceptual understanding of the major concepts in middle and high school mathematics. I then asked whether the little amount of mathematical discourse observed in the classroom could have possibly stemmed from their possession of a stronger procedural knowledge base than a conceptual knowledge base with respect to mathematical concepts. The second part of this question asked them to state whether they believed the lack of discourse could have resulted from their lack of engagement with the mathematics or the mathematical tasks.

Writing prompt

A third piece of instrumentation used to gather data was a writing prompt (See Appendix E) designed to assess what mathematical information the prospective teachers had gained based on their discourse during the class. The prospective teachers completed the prompt through an online survey website, Survey Monkey, and contained two parts. The first part of the prompt asked the prospective teachers to select their level of participation in the discourse. They had the option of five statements. The choices included:

1. I learned a math idea from someone based on our discussion.
2. I taught someone a math idea through my conversation and experiences.
3. I looked at a math idea in a new way based on our discussion.
4. I disagreed or questioned an idea in our discussion today.
5. I referred to someone else's idea from research.

The second part of the prompt asked the prospective teachers to elaborate on their selection encouraging them to explain and justify their selection through specific examples. For example, if a prospective teacher selected that she had disagreed or questioned an idea, then I hoped that she explained the idea she specifically disagreed with and why she questioned the idea.

Data Sources

In order to collect rich, meaningful data and to triangulate the data for dependability and credibility, I utilized four data sources. These data sources included documents, field notes, audio recordings, interview responses and writing prompts.

Documents

One data source utilized was the collection of relevant documents. Documents are a useful source of data because they are exact, readily available, and unobtrusive to the participants (Marshall & Rossman, 1995; Merriam, 2009; Yin, 2003). For this study, I collected all written documents pertaining to the instructional tasks used in the class. These documents included the class syllabus, mathematics worksheets, and any professor-created or prospective teacher-created documents.

On the first night of my observation, the class participated in a mathematical task involving the creation of the unit circle (See Appendix G). The task contained directions and a copy of the unit circle. The purpose of collecting written documents of the instructional tasks was to gain a contextual understanding of the basis of the discourse that occurred during each class meeting. The documents collected also were useful in analyzing whether the questions within the instructional tasks were at a low or high cognitive level. I also wanted to compare and contrast the questioning that occurred *about* the task during the class to the questions, designed for students, *within* the task.

A second document source was the academic transcripts of four of the participants. The purpose of the transcripts was to analyze the prospective teachers' past mathematical experiences. Specifically, I wanted to view the type of mathematics classes each one had taken before the methods course along with the grade they had received in the classes. Additionally, I also wanted to view their Graduate Record Examination (GRE) quantitative scores. The GRE is a standardized test that evaluates one's readiness for graduate school. The GRE measured one's verbal and quantitative reasoning along with analytical writing skills. The GRE quantitative score gives an overall measure of the prospective teachers' mathematical skills among a range of topics such as geometry, algebra, and data analysis as compared to other graduate students. Four of the seven prospective teachers granted me permission to view their transcripts. Three of the seven prospective teachers did not respond to my repeated requests for transcript access.

Participant Observations

A third data source utilized was direct observations. Direct observations allowed me, as the researcher, to witness the phenomenon studied in action in its own setting and in real time. Yin (2003) noted that observations provide contextual data of the participants in their natural setting. Participant observations "allow the researcher to see things firsthand and use her own knowledge and expertise in interpreting what is observed" (Merriam, 2009, p. 119). She pointed out that a challenge a researcher faces with using observations is that they are time consuming. The researcher has to make time to visit the setting and observe the participants in action. In addition, the lack of authenticity has the possibility to exist within the actions and interactions of the participants because they are being observed by an outsider.

For this study, I observed the participants on three occasions as they attended the mathematics methods class. I created an observation protocol (See Appendix C) to guide the observations of the classroom discourse so that the observations were purposeful and the field notes gathered were rich in detail. Observations focused on the structure, nature and purpose of the discourse within the class. Specifically, I gathered field notes on the instructional task observed including the categorization of the task as defined by Shulman (1986) and Hill et al. (2008), as well as a description of the task. Additionally, I noted the type of discourse that took place during the tasks, and the level of communication that occurred based on Brendefur and Frykholm's (2000) classification of communication levels. I noted interactions–a prospective teacher volunteering a response, the professor asking a question about mathematical content, or the prospective teachers talking to her peers about mathematics or pedagogy.

Along with field notes within the observation protocol, I also made notes to myself during the observation. I observed the participants' body language during a task-related discussion and I observed if they were doing other activities, such as using their computer during the class or more specifically, during the class discussions. I also noted questions that I asked myself during the observations. These questions included "Why are the participants not talking?" and "How does the professor know that the prospective teachers understand how to teach the task to students? Where is the talk about teaching the task?" These questions were for my own purpose of reflection as I analyzed the data.

Audio recordings

Along with the participant observations, I recorded each class meeting and transcribed the discourse in order to have written record of the conversations that occurred. The use of the digital recorder did not influence the nature, pattern or function of the discourse. The presence of the digital recorder has the potential to influence what person says (Marshall & Rossman, 1995; Merriam, 2009). Merriam (2009) stated that researchers "find that after some initial wariness, respondents tend to forget that are being taped…" (p. 109). Classroom observations could not adequately capture all of the verbal interactions within a class session. I felt that because the classroom discourse was the focus of study, it was imperative that I had a record of the participants' spoken words during their discussions on the mathematical tasks and pedagogical issues. The recordings ensured that I had access to the full discussions of each class meeting. The recordings of the class meetings allowed for increased dependability when analyzing the discourse. These taped conversations provided a third data source that aligned and connected with the classroom observations. The use of a digital recorder was unobtrusive to the participants as I was sitting in a corner of the conference room with the recorder out of sight. I did inform the participants at the beginning of the data collection that I would be using a digital recorder to record their discussions. None of the participants signified to me that they were uncomfortable with the recording of the discourse.

Writing prompts

A fourth data source was writing prompts as a type of open-ended questionnaire. The questionnaire, as a writing prompt, has the potential to provide rich detail that could be lost in observations (Yin, 2003). The writing prompt used in this study asked the prospective teachers to reflect on their mathematical discussions.

Identical writing prompts were used after all the observed class meetings. The prompt asked the prospective teachers to select how the discussion had affected their understanding or thinking about mathematics. The prospective teachers had five statements from which to choose. The prospective teachers completed the prompt using an online survey website, Survey Monkey. They selected their choice of statement from a drop-down menu below the question. Below the selection, the prospective teachers supported their statement selection in the form of a paragraph. This allowed them to include specific examples from the class to support their selection. I asked the prospective teachers to complete the survey within 24 hours of the class meeting.

The online system allowed them to complete the writing prompt anonymously and within a timeframe that was conducive to their schedule. This timeframe prevented them from feeling rushed to complete the prompt after class. I was hopeful that a flexible time frame would promote placing thought toward the response. The purpose of the prompt was to ascertain the level of involvement and internalization of the discourse for each prospective teacher on their mathematical thinking and understanding.

Interviews

The final data source utilized in this study was interviews. Research has cited the effectiveness of capturing rich, descriptive information from participants themselves (Bogdan & Biklen, 1992; Marshall & Rossman, 1995; Merriam, 2009; Miles & Huberman, 1994; Patton, 2002; Taylor & Bogdan, 1984). Denzin and Lincoln (1994) declared that the interview is the favorite methodological tool of the qualitative researcher. Merriam (2009) described interviews as a conversation with a purpose. Patton (2002) described the purpose of interviewing as the entrance into another person's perspective. Creswell (2007) suggested multiple ways in which interviews can be conducted such as face or telephone interviews as well as interview via electronic mail.

For this study, the decision to conduct interviews was made after the initial data collection and analysis was completed. As I analyzed the data, educated hunches emerged that needed data for validation (Merriam, 2009). Therefore, the need arose to question the prospective teachers using a protocol that was semi-structured in nature. Therefore, four months after the course had ended, I created an interview protocol. The questions I asked were pre-determined; however, based on the answers provided to the questions, additional questions that were not pre-determined might have been asked.

Electronic mail was used to conduct the interviews. Creswell (2007) suggested using electronic mail as a method to conduct an interview. I sent a request for interview participation to all seven prospective teachers on three occasions. However, only three prospective teachers responded to my invitation and stated that they were willing to participate. I emailed each of them a copy of the interview consent form and the three prospective teachers responded with their consent. I then emailed the interview protocol to each participant.

The interview protocol contained six structured questions whose purpose was to gain information on specific topics related to the themes that had emerged from the initial data analysis. For instance, I asked each of three participants to explain why they thought there was more discourse when discussing pedagogical topics than mathematical tasks. I received my first interview response the same day. From reading her responses, I developed a set of four follow-up questions that I sent to her and subsequently, decided to send to all three participants. Within a week, I had received responses to all ten questions from the three participants.

Merriam (2009) noted the complexities that can occur between the interviewer and the respondent when conducting interviews including the way the interviewer states questions and establishes ease between the interviewer and the respondent. Creswell (2007) stated that telephone or electronic mail methods eliminate interviewer influences toward interviewees. I felt that conversing with the three participants through electronic mail alleviated these complexities. I also felt this method established a neutral climate for the prospective teachers because we were not face-to-face. It also avoided any unease the three prospective teachers may have felt in an interview environment.

Trustworthiness

Marshall and Rossman (1995) asserted, “All research must respond to canons that stand as criteria against which the trustworthiness of the project can be evaluated” (p. 143). They defined these canons as questions that ask about how credible, transferable, applicable, and replicable the findings are of the study. A study’s trustworthiness is assessed by these characteristics. Creswell (2007) noted that various qualitative researchers have provided their own terms and stances on trustworthiness. These range from applying new terminology to established phrases to disregarding the idea altogether. Creswell emphasized the need for researcher autonomy in selecting terminology, use, and strategies. Lincoln and Guba (1985) adopted new terminology to describe aspects of the trustworthiness of qualitative research. These new terms included credibility, transferability, dependability, and confirmability. This study utilized the terminology adopted by Lincoln and Guba to describe the different methods of validation.

Credibility

Marshall and Rossman defined credibility as “the demonstration that the inquiry was conducted in such a manner to ensure that the subject was accurately identified and described” (1995, p. 143). Merriam (2009) defined it as the ability to match research findings to reality. Creswell (2007) suggested that a strategy for establishing credibility is to provide a detailed, in-depth description using data of the core issue of the study and all of the variables that comprise the issue. These include the setting and the participants. Providing a detailed description of the nature, pattern, function, and purpose of the classroom discourse in a mathematics methods course aided the study's credibility.

A second strategy I used to establish credibility was member checking. Member checking occurs when the researcher solicits the participants' views of the credibility of the findings and interpretations (Creswell, 2007; Merriam, 2009; Miles & Huberman, 1994). Lincoln and Guba (1985) listed member checking as the most effective strategy for establishing credibility. In this study, member checking occurred to establish credibility of the findings. I asked the professor of the course, who was also a participant in the study, to review the study's findings. I sent the professor my initial draft of the findings through electronic mail. Her review of the findings ensured that what I saw and heard was realistic and reported accurately. The professor stated that she did not have any revisions to the written report (Member checking email, 10/1). She reported that my findings were accurate and consistent with what had occurred during each class meeting.

Looking for alternative explanations was another strategy used to establish credibility in the findings. Researchers refer to this strategy as the negative case analysis (Merriam, 2009; Patton, 2002). Patton (2002) argued, "Failure to find strong supporting evidence for alternative ways of presenting the data or contrary explanations helps increase confidence in the original, principal explanation you generated" (p. 553). As I analyzed my field notes, audio recordings, interview responses, and writing prompt data and began to develop themes, I formulated a proposition regarding the level of engagement of the prospective teachers. The interview protocol addressed themes associated with my proposition with the hope of gaining information that further validated my proposition or provided a counter-argument. I hypothesized that the level of engagement of the prospective teachers was lower during mathematical tasks than during pedagogical discussions due to their high mathematical confidence. In order to establish confidence in my proposition, I searched for an alternative explanation that explained the nature of their engagement. Based on the data and research, I discovered an alternate explanation that accounted for the fluctuation in engagement level that Chapter Five discusses.

Transferability

Researchers have defined transferability as the ability to apply the findings of one study to another context (Lincoln & Guba, 1985; Marshall & Rossman, 1995; Merriam, 2009). Marshall and Rossman (1995) compared transferability with the ability to generalize. They noted that transferability has historically been problematic in qualitative research. However, they emphasized the use of theoretical frameworks to apply generalizations to different contexts as a strategy to establish transferability. The use of established theoretical frameworks related to classroom discourse provided transferability for this study. By viewing the study's findings of this through the lens of Brendefur and Frykholm's (2000) levels of communication and Hill et al.'s (2008) framework for mathematical knowledge for teaching, the use of these findings in other contexts such as traditional teacher education graduate programs or mathematics programs could potentially occur. The findings from this study have the opportunity to be useful in other areas of teacher education in terms of essential teacher knowledge by looking at the findings of this study through the theoretical perspective provided by Hill et al. (2008).

Creswell (2007) noted the use of "rich, thick descriptions" as a transferability strategy (p. 209). Merriam (2009) described rich, thick descriptions as "a description of the setting and participants of the study, as well as a detailed description of the findings with adequate evidence presented in the form of quotes from participant interviews, field notes, and documents” (p. 227). This study provided rich descriptions of the setting and participants along with the findings through quotes, excerpts from audio transcriptions, and numerical tabulation data to support its credibility and transferability.

Triangulation

Triangulation involves using multiple data sources, methods, and theories in order to corroborate findings (Creswell, 2007; Marshall & Rossman, 1995; Merriam, 2009; Miles & Huberman, 1994; Patton, 2002; Yin, 2003). Patton (2002) explained the basis of triangulation as the notion that no single method ever adequately solves the problem of rival explanations. Patton also noted that triangulation allows the researcher to view data in diverse ways.

Triangulation using multiple sources established credibility for this study. These multiple sources included observation field notes, document collection, and audio recordings of classroom conversations, written prompts, and interview responses. I used triangulation to establish whether themes were consistent across multiple sources. For example, when I analyzed the participant observation data I had collected, I noticed that the discussions regarding the mathematical tasks did not have a strong focus on teaching methods and mathematics pedagogy. Therefore, I looked to my other data sources, such as the audio recording transcriptions and the interview participant responses, to determine whether there were data to support this claim.

Triangulation can be achieved by using multiple methods. Patton (2002) described this process as comparing data collected through qualitative methods with data collected using quantitative methods. This study did not incorporate any quantitative measures. However, numerical data were generated from the data sources. The numerical data attempted to establish compatibility between the data sources and the themes; thus increasing credibility. In this study, I compared quotes, observation notes, and audio transcriptions with the numerical data I generated from the same sources regarding the participants' frequency of verbal occurrences. Additionally, I employed the same comparisons to questioning. I compared the qualitative data about questioning to the numerical data generated that addressed the participants' frequency and level of questions.

Dependability

Lincoln and Guba (1985) equated dependability in qualitative research with reliability. Merriam (2009) defined reliability as the extent to which research findings can be replicated. In qualitative research, replication has the potential to be problematic due to the human nature. Individuals from one study to the next are different with different characteristics, backgrounds and behaviors. (Marshall & Rossman, 1995; Merriam, 2009). Therefore, dependability involves determining whether the results of the study are consistent with the data collected. Merriam (2009) asserted that "rather than demanding that outsiders get the same results, a researcher wishes outsiders to concur that, given the data collected, the results make sense" (p. 221).

The implementation of specific, research-based strategies aid in a study's dependability. Merriam (2009) included triangulation as a strategy. Miles and Huberman (1994) advocated relating to the reader the specifics of the data collection and data analysis process. Lincoln and Guba (1985) referred to this as an audit trail. Specifically in case study research, Yin (2003) referred to this as a case study protocol. My case study protocol provided detailed procedures in the collection and analysis of data. Case study protocols include descriptions of the data collection and analysis including specific information such as codes used and category development. Merriam (2009) noted that this detailed account is typically in the methodology section of a study.

Data Analysis

Marshall and Rossman (1995) characterized data analysis as the process of bringing order, structure, and meaning to the mass of collected data. They also noted, “Qualitative data analysis is a search for general statements about relationships among categories of data” (p. 111). Merriam (2009) described data analysis as the process of making sense of one’s data. She noted that qualitative data analysis is “*inductive* and *comparative*” (p. 175). Inductive analysis involves beginning with specific data and from that data, moving outward to establishing emerging themes, testing those themes, and then expanding them into general statements. Comparative analysis involves comparing one segment of data with another to determine similarities and differences. Thus, the researcher is looking for patterns. I utilized both of these methods in the data analysis for this study. Marshall and Rossman (1995) separated analytic procedures of data interpretation into different modes: organizing the data; generating categories and themes; testing the themes against the data and searching for alternative explanations.

Organizing the data

Merriam (2009) suggested that the process of data analysis begins with identifying segments of data that are responsive to the research questions. Identifying useful segments of data eliminated extraneous data. Marshall and Rossman (1995) noted that the researcher would become familiar with the data through this process in an intimate way. The initial data reduction involved reading through the data several times. This included reading through my field notes from my participant observations, the content of the mathematical tasks, the audio recording transcriptions, the responses to the written prompts, and the responses to the interview questions. Through this reading, I was able to reduce the data and develop units.

Merriam (2009) defined a unit of data as any meaningful segment of data. For example, the audio recording transcriptions contained all of the discussions that occurred throughout each of the class meetings. The participants spoke about various different topics. Some of the discussions did not relate to any of my research questions such as the professor discussing the manner in which she had graded the prospective teachers' homework assignment or the prospective teachers discussing their plans for the weekend. Therefore, I excluded these types of discussion from my data. The resulting units of data contained discourse related exclusively to the course and its content.

Another example of identifying units of data occurred in the document analysis. In the document analysis, I specifically focused on the questions that were contained in the mathematical tasks. Therefore, I reduced the data within the documents to only the questions of the task by highlighting those relevant questions. From this analysis of all the data sources, I formed units of data. All of the identified units from the data formed the case study database (Merriam, 2009; Yin, 2003).

Coding

Patton (2002) defined coding as the process of classifying data. Coding allows the researcher to find, pull out and cluster segments of data that relate to a research question, proposition or theme (Miles & Huberman, 1994). They defined codes as "tags or labels for assigning units of meaning to the descriptive or inferential information compiled during a study" (Miles & Huberman, 1994, p. 56). Creswell (2007) compared coding to a spiral in which the researcher is frequently assigning and reassigning characteristics to the data stating, "In this process, I finally come to a point at which the categories are 'saturated'; I no longer find new information that adds to my understanding of the category" (p. 240). Coding is abbreviations that allow the researcher to identify sentences that represent ideas or concepts that relate to the study.

Open Coding. Open coding consisted of line item coding of the units of data from the audio recordings, interview responses, and observation field notes for each class meeting. For this study, I used a coding scheme devised by Nathan and Knuth (2003) as they studied the discursive patterns of a middle school teacher over a two-year period, to characterize the direction of the participants' discourse. Creswell (2007) suggested being open to adding codes to an established coding scheme. I did add additional codes to my coding scheme that addressed the forms of mathematical knowledge for teaching.

As I began coding, I labeled the direction of the discourse TS if it was from the professor to a prospective teacher, ST if the direction was from the prospective teacher to the professor or SS if the flow of direction was from prospective teacher to prospective teacher. If the professor asked a mathematical question to the whole class, it was labeled QM . If the nature of the discourse revolved around mathematical content knowledge, the unit of data was labeled MK. Additionally, if the function of the discussion was a response by a prospective teacher to a mathematical question asked by the professor, then it was coded RM for response to mathematics question. The open coding of the units of data resulted in more than one code assigned for a unit of data. The units of data contained two to three codes that described the flow of the discourse, the teacher knowledge that was addressed, and the function of the discourse. The codes within the discourse-coding scheme that were used during the open coding process are reflected in Table 2.

Table 2

Discourse Coding Scheme

Code	Description
TS	Professor to Prospective teacher
ST	Prospective teacher to Professor
SS	Prospective teacher to Prospective teacher
TC	Professor to Whole class
QM	Asks math question
QP	Asks pedagogy question
MS	Math statement-reiteration of facts, rules, process, etc.
RM	Response to mathematics question
RP	Response to pedagogy question
CK	Content knowledge
PK	Pedagogical knowledge
CuK	Curricular knowledge

My first research question involved determining the role of questioning within discourse. Researchers have suggested that questioning plays a prominent and powerful role in facilitating classroom discourse. Therefore, I developed an additional coding scheme to classify the type of questions that occurred in the data. I created this coding scheme from the cognitive process dimensions of questioning developed by Anderson et al. (2001). The codes represent the level of cognitive process that the question required. The codes developed for the questions that occurred during the classroom discourse are reflected in Table 3.

Table 3

Questioning Coding Scheme

Cognitive Process	Code
Remember	QR
Understand	QU
Apply	QA
Evaluate	QE
Create	QC

Within each unit of data, I coded the questions asked by the participants. Additionally, I coded the questions embedded within the mathematical tasks. For example, the question “How many degrees are in a right angle?” was coded QR because it was a question that asked for recall of information.

I used two methods to complete the open coding. Initially, the coding was completed by hand. Codes were written in the margins of each unit of data. As each unit was coded, relevant words, phrases, or sentences were highlighted along with any questions that were asked. These key phrases and questions were listed in an Excel spreadsheet that contained columns identified as each of the open coding nodes.

Toward the completion of the hand coding, NVivo software was incorporated to aid in data analysis. Therefore, open coding was performed again using NVivo. I imported the data into the software and established my units of data by eliminating sections of the data that had already been established as irrelevant. Then, I listed my discourse coding scheme, and questioning code scheme as free nodes. Nodes were the term used in NVivo for codes. In NVivo, free nodes are stand-alone nodes that do not have a relationship with any other nodes. Since I was using open coding to code initially all of my units of data, I listed all of my codes as free nodes. Using the same process as hand coding, the units of data were coded under the free nodes. I highlighted the text that I wanted to code and then selected the coding node that I wanted to use to code the data. If I selected a node, all of the units of data that were coded under that node appeared.

Axial coding. Axial coding is the process of grouping codes into emerging themes or explanations (Merriam, 2009). Marshall and Rossman (1995) described axial coding as sorting objects based on like attributes such as grouping fruit based on color. Axial coding is the next step in the coding process once open coding has been completed.

For example, during axial coding, units that were coded for the flow of direction were grouped into similar concepts. If a verbal interaction was between the professor and a prospective teacher and the interaction originated with the professor, it was coded TS for professor-to-prospective teacher. Likewise, a verbal interaction was coded SS for prospective teacher-to-prospective teacher if the verbal interaction was horizontal. From these free nodes, two parent nodes were established. Parent nodes, in NVivo software, were broader nodes in which other nodes can be grouped. The parent nodes, vertical discourse and horizontal discourse, were established. Then, the TS and ST nodes were grouped under vertical discourse and the SS node was grouped under the horizontal discourse node. The parent node, vertical discourse, contained any data that had been coded as TS or ST. The parent node, horizontal discourse, likewise, contained any data that had been coded as a peer interaction.

An additional example of axial coding included the coding of the free nodes for questions. Based on the research from questioning in Chapter Two on Anderson et al.'s (2001) framework, questions were categorized as low-level and high-level. Low-level questions were categorized as *remember* and *understand* questions and high-level questions as *apply*, *evaluate*, and *create* questions. Therefore, two parent nodes, low-level and high-level questions, were created and the *remember* and *understand* nodes were grouped under low-level questions and the *apply*, *evaluate* and *create* questions were grouped under high-level questions. Once axial coding was completed based on the free nodes, there were seven larger categories of data. These categories are listed in Table 4.

Table 4

Discourse Data Axial Codes

Parent Code	Code
Vertical discourse	VD
Horizontal discourse	HD
Analytical scaffolding	AS
Low-level questions	QL
High-level questions	QH
Mathematical knowledge	MK
Pedagogical knowledge	PK

Selective coding. Selective coding developed from grounded theory. In fact, Merriam (2009) noted that aspects from grounded theory are used throughout qualitative research. Selective coding involves the process of identifying core categories. Selective coding is used to code data to a core category, proposition or hypothesis (Merriam, 2009).

There were core categories that had developed from the completion of open and axial coding. These core categories consisted of the pattern, function, nature, and the prospective teachers' engagement in the discourse. For example, the pattern of the discourse included the directional flow of the discourse, vertical or horizontal. Therefore, I merged the axial nodes, vertical and horizontal discourse, into a core category labeled, *pattern of the discourse*. I also noted that the topics that formed the basis of the conversations were either mathematical or pedagogical. The mathematical tasks and pedagogical discussions served as the contextual setting for the classroom discourse. Thus, those selected nodes were coded under the core category, *nature of the discourse*.

Creating and testing emerging themes

Once I had completed the axial and selective coding process, themes emerged from the data that represented potential findings. For example, when I reviewed the data under the core category, *nature of the discourse*, I noticed that the questions that were asked were being used to assess understanding of mathematical concepts. Thus, questioning as a form of assessment became an emerging theme. The data from the core category, *pattern of the discourse*, showed that vertical interactions dominated the discourse. Therefore, the dominance of vertical discourse became an emerging theme.

The next step from creating emerging themes is to test the themes against the data (Marshall & Rossman, 1995; Merriam, 2009; Miles & Huberman, 1994; Patton, 2002). Triangulation allows the researcher to test themes against the data. By testing themes across multiple sources, the data substantiates or refutes emergent themes. For example, an emergent theme was *mathematical tasks yielded limited discourse due to a lack of engagement*. To test this theme, I analyzed my field notes that referenced the environment and body language of the prospective teachers. I also examined the audio recordings specifically focusing on the responses from the participants during the mathematical tasks. Finally, I reviewed the interview participants' responses that related to the lack of discourse in the mathematical tasks. The data sources supported the theme.

Some of the subcategories were not supported by the data and therefore, not developed into themes. For example, I made the proposition from my observations that field experience was a contributing factor to the type of discourse that occurred during the class. I strongly believed this proposition initially. However, when I tested the proposition against the data I had collected, the data did not support my claim. The themes that were generated from supporting evidence represented an important finding toward answering the study's research questions.

I also tested my themes by looking for alternate explanations (Merriam, 2009). Alternate explanations involve searching for another reason that the emerging themes appeared in the data. Using a negative case analysis provides increased credibility to a study's findings (Merriam, 2009; Patton, 2002). Using the previous example, I further hypothesized that the prospective teachers' lack of engagement toward mathematical tasks resulted from a high mathematical confidence, their self-efficacy. I looked at my data and to the literature to determine whether an alternate explanation existed for their lack of engagement and discourse. Examining multiple data sources, including field notes, audio recording transcriptions, writing prompts, and interview responses, I searched for another explanation that would refute my proposition. Another explanation for the lack of engagement was not found within the data but was found in the literature. Therefore, while my proposition was strongly supported by the data, there was literature that offered an alternate explanation and that explanation was included in my analysis of findings.

Along with the themes established from the qualitative analysis, numerical data were generated to determine the frequency of the verbal interactions of the participants within the discourse that occurred for each class meeting. The numerical data were generated from the audio recording transcriptions. Within each unit, all of the verbal statements made by the professor were tabulated along with the number of verbal statements made by the prospective teachers. The same procedure was performed for questions. I tabulated the number of questions that were asked by the professor and the prospective teachers. Additionally, I tabulated the number of each type of question that was asked according to the cognitive level of questioning developed by Anderson et al. (2001). Since the data showed that there were two main types of discourses, namely mathematical and pedagogical, I calculated the frequency of verbal interactions for each type separately. Through hand counts from the identified units of data, the frequency of questions asked, responses given, and explanations provided by the participants were tabulated and used to test and support the findings.

Limitations

A limitation to the study included the number of participants. The sample consisted of seven prospective teachers and one professor. The small sample limited the transferability of the data to other populations. The sample also proved to be a limitation in the implementation of the written prompts. Out of the 21 possible total prompts to be completed, I only received 13 prompts. After the first observed class, five prompts were completed. After the second and third observed class, four prompts were completed each time. Additionally, the depth of the responses offered little insight into their thinking. Their responses about their discourse participation were short and not descriptive.

Another limitation to the study included the timing of the data collection. I began my data collection during the middle of the semester. I was not able to observe the nature of the discourse that occurred at the beginning of the semester. Therefore, the data were disjointed, not cohesive. Additionally, the interviews were conducted four months after the mathematics methods course had ended. The three prospective teachers who participated in the interviews had to retrieve their experiences from the class from memory. Therefore, the details and the richness of their responses might have been different if the interview had been conducted during the class or shortly after the conclusion of the class.

The Role of the Researcher

Merriam (2009) described the role of the researcher as one whose role is to understand how people interpret their experiences. Researchers have accomplished this in different ways. In this study, my role as a researcher was that of a participant observer.

My purpose was to observe how the participants engaged in the discourse in their mathematics methods course. I was not directly involved in the discourse although there were instances in which I participated in the discourse. These instances were few and not enough to include myself as a participant.

CHAPTER IV

RESULTS

The purpose of this study was to examine the discourse that occurred within a graduate mathematics methods course. Specifically, this study has sought to examine the characteristics of the discourse among prospective mathematics teachers and the type of teacher knowledge the discourse facilitated in their mathematical and pedagogical development. During the classroom observations, the prospective teachers and professor participated in tasks and discussions that were either mathematical or pedagogical. Pedagogical discourse revolved around the philosophy of teaching or teaching strategies. Mathematical discourse focused on specific mathematical concepts and tasks. These two categories displayed differences in the pattern and engagement of discourse. Hence, I find it worthy to provide a contextual setting for both types of discourse.

During my classroom observations, the class participated in three mathematical tasks and three microteaching lessons. These mathematical tasks were designed for the middle and high school classroom. The first observed task focused on data collection, analysis and interpretation. For this task (See Appendix F), the prospective teachers collected data, then analyzed the data by calculating measures of central tendency, and interpreted the data in order to make conjectures. For the second task (See Appendix G) I observed, the prospective teachers derived and replicated the unit circle and then applied this derivation to the creation of a sine graph. The third mathematical task I observed

(See Appendix H), focused on building polynomial functions and the prospective teachers' exploration of the functions using a graphing calculator. The microteaching lesson task consisted of a prospective teacher assuming the role of a teacher and teaching a mathematics lesson to the other prospective teachers who assumed the role of students. During my observation, I observed three prospective teachers teach lessons to the class. Lisa presented a lesson on volume and surface area of rectangular prisms (See Appendix I). Jill presented a lesson on determining theoretical probability (See Appendix J). Amy's lesson (See Appendix K) targeted the concept of factoring. All of the tasks involved engagement strategies such as manipulative and technology use. Each task also incorporated written comprehension and application questions that connected the task to the mathematical concept being investigated.

Along with mathematical content, pedagogy was an instructional priority as demonstrated by its inclusion in the course syllabus and the time allotted each class meeting to its discussion. All of the eight goals for the course listed in the syllabus embedded pedagogical knowledge and practices (Document 1, 3/26). Such goals related to management, assessment, and instructional strategies. The prospective teachers were assigned readings that focused on various pedagogical topics and wrote reflections on the readings that were submitted to the professor. The readings were discussed during the class; however, the reflections were not shared during class. The prospective teachers submitted their reflections to the professor and the professor provided feedback to them. In doing so, another form of vertical discourse was promoted as the reflective conversation was only between the professor and each prospective teacher. During my classroom observations, the prospective teachers and the professor participated in several discussions regarding pedagogy. In one class meeting, they engaged in a discussion over the quality of an essential question. An essential question is a question that teachers develop that guides student learning. Another discussion occurred as the professor presented information on the various modes of instruction. These modes of instruction included lecture, question and answer, discovery learning, laboratory learning, and collaborative work. A third discussion that was observed involved the prospective teachers and professor in a lively conversation over discipline, which led to the topic of homework. Other discussions involved topics such as note taking and copy paper.

Within this chapter, four significant themes that emerged from the analysis of the data are discussed. The themes that resulted from the analysis of the data represent not only answers to the study's research questions, but also provide an insight into the conversations within a graduate secondary mathematics education methods course. The four themes that emerged from the data were the dominance of vertical discourse, the prominence of questioning in directing and sustaining discourse, the use of scaffolding to deliver mathematical and pedagogical knowledge, and the influence of engagement in shaping discourse.

The Dominance of Vertical Discourse

Hatano and Inagaki (1991) defined vertical discourse as verbal interactions that occur between an individual who possesses more knowledge about a topic and an individual that lacks the same knowledge. With vertical discourse, a hierarchy exists that is defined by depth of knowledge or traditional educational roles such as teacher and student, or in the context of this study, professor and prospective teacher. When vertical discourse is defined by roles, it is assumed that the teacher possesses more knowledge and experience about a topic or concept than the student does. Hatano and Inagaki defined horizontal discourse as discourse that occurs among individuals that are presumed to be at the same knowledge level such as students in a class or in this case, the prospective teachers. In this study, vertical discourse dominated the directional flow of the interactions of the participants. Its pronounced existence occurred during discourse surrounding the mathematical tasks. When the discourse focused on pedagogical topics, the interactions of the prospective teachers began to develop characteristics of horizontal discourse.

The Vertical Interactions Between The Professor and Prospective Teachers During Mathematical Discourse

The observed mathematical discourse pattern of this graduate mathematics methods course was similar to the discourse pattern found in many middle school and high school mathematics classes. The discourse mirrored the following pattern: teacher questions, student answers, teacher comments, and the teacher asks another question (Sinclair & Coulthard, 1975). Table 5 displays the verbal interactions between the professor and the seven prospective teachers when the topic of discourse was a mathematical task. The first column, *Asking a Question*, represents all the questions that were asked during the mathematical tasks. The second column, *Providing a Response*, represents when a response was provided to a question or a response was provided to another response. Therefore, comments that were mathematical in nature were included in this category. Finally, the third column, *Giving an Explanation*, represents expanding thoughts, restatements, or summarizations provided about a mathematical concept, procedure, or strategy. In some instances, more than one of these categories were embedded together. For example, an explanation might have had a question attached to it such as, “Yes, it depends on what it is you’re trying to get your students to see. A histogram does not compare individual sets. Ok, interesting. Could you have, and this is just a question that I’m throwing out there, could you have put these numbers along your horizontal?” (Audio recording transcription 1, 3/26). The data from Table 5 reflects the pattern of the discourse between the professor and prospective teachers and the dominance of the question-and-answer format. Questioning comprised 52% of the verbal statements made by the professor while providing a response comprised 66% of the verbal statements made by the prospective teachers. Table 5 does not include any comments made that were not mathematical in nature such as “All right. Let’s do it. Make it pretty” (Audio recording transcription 1, 3/26).

Table 5

Frequency of verbal interactions during the three mathematical tasks (excluding microteaching lessons)

	Asking a question	Providing a response	Giving an explanation
Professor	78	17	56
Prospective teachers	17	75	22

When the discussion revolved around a mathematical task, Table 5 affirms that the professor was in control of the conversation. Within the three mathematical tasks observed, the professor made 151 mathematical verbal statements compared to the 114 statements made by the seven prospective teachers. The professor asked four times as many questions than the prospective teachers asked. During the three mathematical tasks, the professor asked 78 questions compared to the 17 questions asked by the prospective teachers. She also provided twice as many explanations. The professor asked many questions regarding the mathematical tasks and the responses generated from the prospective teachers represented answers that were often rote. They did not explain their thinking on their own. Typically, an explanation required another prompt or question from the professor. This continued probing generated the 22 explanations provided by the prospective teachers. It also contributed to the great amount of questioning by the professor. Overall, the communication within the mathematical tasks resembled a vertical pattern of traditional, teacher-centered discourse. An example of the vertical discourse within the class occurred during the completion of a task involving the derivation of the unit circle and sine graph.

Professor: There is a way we can derive [the unit circle], right? How can we do that? With what kind of triangles?

Prospective Teacher 1: Right.

Professor: Or?

Prospective Teacher 1: Special right.

Professor: Well, no, I actually want you to derive it from scratch. Let’s use the equilateral triangle, what does it mean about the angles?

Prospective Teachers: They’re all the same.

Professor: How many degrees in here?

Prospective Teachers: 180.

Professor: So each one of these has to be?

Prospective Teachers: 60.

Professor: Very good. Are you with me? And what do you know about all the sides?

Prospective Teachers: They're all equal.

Professor: They are. I'm just going to pick two. It doesn't matter what you pick as long as they're all the same, correct? Now, I'm going to drop a perpendicular and when I drop that perpendicular, what do I know about this?

Prospective Teacher 1: Right angle, 90 degrees.

Professor: Yes. And what do I know about this little angle right here?

Prospective Teacher 2: It's half of 60, 30.

Professor: Which is 30. Ok, so this, I'm going to take this and I'm going to pull it out and put it over here so I can see it better. So I have 90 here, I have 60 here, and I have 30 here. This is still going to be 2, do you agree?

Prospective Teachers: Yes (Audio recording transcription 1, 3/26).

During this interaction, the professor asked the prospective teachers questions about an equilateral triangle. Through her specific questioning, the professor assessed whether the prospective teachers had an understanding of the properties of an equilateral triangle so that they could derive the unit circle and subsequently, explain the derivation to high school students. She evidenced this through her questioning, questioning that solicited explicit knowledge of triangles. She also frequently asked the prospective teachers questions such as "Do you agree?" and "Are you with me?" Throughout the questioning, the professor looked for specific answers that demonstrated comprehension and prior knowledge. The responses the prospective teachers provided demonstrated their procedural knowledge of an equilateral triangle. However, the professor did not solicit explanations for their responses and the prospective teachers did not provide explanations. She validated their responses and offered explanations through her own response. Therefore, she could not explicitly determine whether the prospective teachers possessed conceptual knowledge of equilateral triangles.

During the same class, the prospective teachers completed a data analysis task in which they collected data based on their participation in a word recall activity and then they used their data to calculate measures of central tendency, such as mean and median (Field notes 1, 3/26; Document 3, 3/26). After the data was collected, the professor assessed whether the prospective teachers knew how to represent the data graphically.

> Professor: So what do you think? What if I wanted you to graph it and present it to a group of teachers? What kind of graph would you use?
>
> Prospective Teacher 1: I would do a bar graph.
>
> Professor: Yeah, anybody else do something else?
>
> Prospective Teacher 2: I actually did the box and whiskers.
>
> Professor: Well it could be, but I think if you did a double bar graph.
>
> Prospective Teacher 1: Yeah, I would do a double bar graph.
>
> Professor: I think the double bar graph would work (Audio recording transcription 1, 3/26).

This conversation between the professor and the prospective teachers represented the majority of the discourse observed during the mathematical tasks because the responses were succinct and without explanation. The prospective teacher, in the above dialogue, did not explain why she would use a bar graph versus another type of graph. Likewise, the other prospective teacher did not explain why she selected to represent her data using a box and whiskers plot. While they communicated their mathematical thought, they did not explain or expand their thinking to demonstrate the conceptual knowledge behind the thought. This short interaction demonstrated that the prospective teachers had different ideas on how to represent the data in the most effective way but did not justify their ideas verbally nor did they expose the other prospective teachers to their thinking regarding their selection. By providing a justification to their response, the prospective teachers had the opportunity to offer a new perspective of viewing the representation of data to the class.

The discourse within the microteaching lessons resembled the vertical discourse observed during the mathematical tasks. Each prospective teacher who led a microteaching lesson-Jill, Lisa, and Amy interacted with the other prospective teachers, who were acting as students, in the traditional teacher-student role (Field notes 3, 4/26). The goal for Jill, Lisa, and Amy was for the class to obtain the correct answers to their questions as was evidenced in their type of questioning. The limited horizontal discourse that occurred during the lessons demonstrated that discourse did not focus on incorporating student talk (Field notes 3, 4/26). However, the prospective teachers commented on their integration of horizontal discourse as an instructional strategy. Amy commented that discussion was the mode of instruction that she most often applies in her classroom. She stated, “I think I use discussion the most. That’s my thing. I like to talk and I like to hear people talk. I like to argue and all kinds of good stuff” (Audio recording transcription 2, 4/12). Lisa remarked on her use of student dialogue stating, “I like doing discussions a lot, but I like to do it without presenting the material first. Like, I want ideas, how do you think we should solve it, how do you think we should go about doing this problem" (Audio recording transcription 2, 4/12).

While the prospective teachers spoke of their integration of horizontal discourse and promotion of strategies, their microteaching lessons did not reflect this integration. The prospective teachers had expressed their fear and frustration with allowing their field experience students to engage in collaborative discussions during class. During a discussion, the professor asked the class their thoughts about students talking in the classroom. She asked if they allowed their students to talk during class. Amy remarked,

> I don't mind conversations, but if it's math conversations. My problem is those kids, they're talking about math and two seconds later, they're talking about the baseball game. They see you walking up, and they are like, "Yeah, so uh." That's what I don't like. If I could trust my kids enough to keep on what we're talking about then I'm fine with that. And as long as they're talking about class, don't make loud noise, you know we're talking in inside voices, and we're doing work…These are kids, so they'll start talking about all kinds of randomness and that's a problem for me. So now, y'all are going to be quiet. We're done talking (Audio recording transcription 2, 4/12).

None of the three lessons I observed incorporated peer dialogue as an element of instruction. Lisa, the first teacher, taught a sixth grade lesson that was designed to teach volume and surface area of solid objects. Her warm-up consisted of a rectangular prism whose length was two meters, width was four meters, and height was four meters.

Lisa: And then plugging things in, what did you get for your first surface area? I'm skipping a couple of steps.

Prospective teacher 1: 32, no 64.

Lisa: 64 and units?

Prospective teacher 1: Meters squared.

Lisa: We do the same process with volume and what did you get for the first volume?

Prospective teacher 2: 32.

Lisa: And the units are?

Prospective teacher 2 Meters cubed.

Lisa: And then we go through, do it for the next section. What was your second surface area?

Prospective teacher 3: 72.

Lisa: 72 square meters. And the volume was once again?

Prospective teacher 3: 32.

Lisa: And the last one, surface area was?

Prospective teacher4: 68, volume 32.

Lisa: So what did y'all notice in doing this warm up?

Prospective teacher 2: All the volumes were the same but the surface areas were not (Audio recording transcription 3, 4/26).

Lisa admitted to the professor and her peers that she found it difficult to take on the role of the teacher and to see her peers as her students (Audio recording transcription 3, 4/26; Field notes 3, 4/26). As Lisa assessed the activating activity, she alternated between questioning the prospective teachers as students and discussing her rationale to them as peers. Lisa had multiple opportunities to invite demonstrations and explanations to the questions she asked. Her discourse was strictly vertical in nature as she asked for answers and solicited responses. She did not ask the prospective teachers to provide an explanation or justification to their responses. Therefore, she took only their answers as affirmation of their understanding of volume and surface area.

The second student teacher, Jill, also mirrored the pattern of vertical discourse. Her lesson on theoretical probability began with a warm-up on fraction, decimal, and percent conversions (Field notes 3, 4/26; Document 7; 4/26). She had listed 10 fraction/decimal/percent conversions on a PowerPoint slide and the class had to convert the given number into another form. Jill questioned the prospective teachers on the warm-up in a manner similar to the way Lisa did.

> Jill: The first one, it says re-write the fraction in simplest form. What did you have for your first one?
>
> Prospective teacher 1: ¼
>
> Jill: Good. Who wants to do number 2? Raise your hand.
>
> Prospective teacher 2: 1/10
>
> Jill: Number 3?
>
> Prospective teacher 3: 1/2
>
> Jill: 1/2 (Audio recording transcription 3, 4/26).

This question-and-answer format continued for the second part of the warm-up on writing a fraction as a decimal or percent. As with Lisa, Jill did not ask for explanations to the answers they gave nor did Jill ask for demonstrations on how to perform the conversion. Table 6 reflects the breakdown of the discourse during the microteaching lessons. The discourse was dominated by the question-and-answer format. The data from Table 6 reflects the strong vertical nature of the discourse and the overwhelming dominance of questioning. Lisa, Jill, and Amy asked 71 questions collectively during the three lessons. They controlled the discussion. Each decided the direction of the discourse. Even though each microteaching lesson contained a task that embedded collaborative work, collaborative discussion of thought and procedures was not promoted. Little horizontal discourse occurred during the microteaching lessons.

Table 6

Frequency of mathematical verbal interactions during microteaching lessons

	Asking a question	Providing a response	Giving an explanation
Teachers	71	5	20
Students	8	66	8

Note. Teachers represent the three prospective teachers who presented a lesson and students represent the prospective teachers who assumed the role of students during the lesson.

During the mathematical tasks and microteaching lessons, the discussions lacked any horizontal discourse. The prospective teachers did not engage in mathematical discussions with each other. Instances in which a prospective teacher responded to the professor’s question or comment and then another responded did occur. Yet, it was interesting to note that the prospective teachers rarely spoke to each other. They directed their verbal statements toward the professor. When they spoke, they faced the professor even if their response was to another prospective teacher's remark (Field notes 1, 3/26; Field notes 2, 4/12; Field notes 3, 4/26). This occurred during my third night of observation during the microteaching lessons. At the end of each lesson, the professor asked the prospective teachers to list strengths and weaknesses for each lesson and then share their comments. Jill taught a lesson on determining theoretical probability. After the lesson, the professor asked the class for their feedback.

> Professor: For Jill, what did you really like? What did you really like about Jill's lesson?
>
> Prospective teacher 1: It was hands-on.
>
> Prospective teacher 2: I thought the entire time she kept relating it back to the lesson on set theory, what we know about discrete variables, what we know about continuous.
>
> Professor: She did a great job doing that, I thought as well. I thought she provided some really great questioning throughout the lesson as she was just circulating the room. How many of you really thought she was very enthusiastic? She has a lot of enthusiasm, doesn't she? (Audio recording transcription 3, 4/26).

When the prospective teachers provided feedback about the lesson, they did not personally address or acknowledge the teacher of the lesson. They addressed the teacher in third person (Field notes 3, 4/26). The conversation occurred between the prospective teachers and the professor *about* the prospective teacher who had taught the lesson instead of with the prospective teacher who taught the lesson. While the professor provided the environment of comfort and safety necessary for horizontal discourse, the prospective teachers did not engage in discourse with each other.

The Integration of Horizontal Interactions During Pedagogical Discourse

The participants met and talked in a safe and comfortable environment. The ease in which the prospective teachers spoke with the professor acknowledged their comfort (Field notes 1, 3/26; Field notes 2, 4/12; Field notes 3, 4/26). They had the opportunity to discuss openly any topic of their choosing even if the topic veered away from the agenda for that class. The professor validated their ideas and comments and responded openly. This deviation occurred on more than one occasion. Such an occasion occurred when Lisa spoke of the amount of Criterion Referenced Competency Test (CRCT) practice she was witnessing in her field experience classroom. The professor responded to her prompt stating, “Okay. We’re going to talk about that for a minute or two. What are you observing there?” (Audio recording transcription 1, 3/26). The professor realized Lisa’s need to voice her opinion and to have her comments listened to and validated. Within this backdrop of safety and ease, the prospective teachers had multiple opportunities to participate in discourse, not only with their professor, but also among themselves.

The discourse surrounding pedagogy was primarily vertical in nature. The professor remained in control of the conversation as she asked questions, offered feedback and directed the flow of conversation. The data reflected in Table 7 demonstrates that the prospective teachers became active participants in pedagogical discourse. The professor made 123 verbal statements while the prospective teachers made 155 verbal statements. However, the professor remained the facilitator of the discourse. The professor asked twice as many questions as they did and also gave twice as many explanations during the pedagogical discussions. Questioning by the professor constituted 45% of her verbal statements. The prospective teachers sharply increased their number of responses which signified their active role in contributing to the discourse. Responses comprised 79% of the verbal statements made by the prospective teachers. Table 7 reflects the verbal interactions between the professor and the prospective teachers during the observed pedagogical discourse.

Table 7

Frequency of verbal interactions during pedagogical discussions

	Asking a question	Providing a response	Giving an explanation
Professor	49	58	22
Prospective teachers	20	123	12

The data from Table 7 depicts prospective teachers who engaged in the discourse within the class when it concerned pedagogy. An example of their active interaction during pedagogical discourse occurred during a side bar conversation on cumulative exams as a checkpoint assessment of student learning.

> Professor: Do you think that if they had, if the students were given cumulative exams that it would be helpful at this point [in the year]?
>
> Prospective teacher 1: But they're having enough trouble with the material you cover in the last week or two, much less everything else that they are supposed to remember.
>
> Prospective teacher 2: In theory, I like the idea of the cumulative assessment to keep it fresh in their minds, but at the same time, some kids really struggle with certain topics but are really good at other topics. If the topic they are struggling with is constantly reappearing, it's going to be constantly dragging them down.

> Prospective teacher 3: I like the thing of using the warm ups though because then they're still seeing them but not in that setting where it's… they're not getting counted off time and time again (Audio recording transcription 2, 4/12).

This interaction typifies several interactions within the discourse regarding teacher pedagogy. In this case, as with other discussions during pedagogical discourse, more than one prospective teacher responded without an additional prompt from the professor. Additionally, the prospective teachers provided responses that were descriptive and contained more than a single response. An interesting note is that the above verbal interaction had the potential to be identified as horizontal discourse among the prospective teachers if they had been speaking to each other. However, the three prospective teachers involved in the exchange did not speak to each other (Field notes 2, 4/12). They directed their responses toward the professor as was noted by the direction of their response and that they were looking at the professor when they spoke (Field notes 2, 4/12).

While the discourse concerning pedagogy still followed a vertical pattern, on several instances, more than one prospective teacher offered an answer to a question or made a comment to an answer, explanation or other comment provided by a peer (Field notes 1, 3/26; Field notes 2, 4/12). Therefore, the discourse was not primarily between the professor and an individual prospective teacher while the other prospective teachers listened. As they were contributing their responses, their responses may or may not have been similar because one correct response did not exist to the questions being asked. During a discussion on various modes of instruction, the professor asked the prospective teachers a question that generated different ideas and viewpoints.

Professor: Do you ever integrate different modes of instruction? If so, how have you done it or how have you seen it done?

Prospective teacher 1: I think laboratory and question and answer often goes well because it's student-centered and they're doing their own activity. And then while they're still in those groups you bring their attention back and you all discuss as a group their findings and have them discuss among groups.

Professor: That's a good example of how you can integrate. What's some others?

Prospective teacher 2: Well, there's laboratory and group discussion. "Why did we do it this way?" and that kind of group discussion, "Why?"

Professor: Good.

Prospective teacher 3: I think even the three, the lab, discussion, and question and answer, because they all fit nicely together and blend well together (Audio recording transcription 2, 4/12).

Instances of horizontal discourse occurred in which the prospective teachers spoke to each other instead of to the professor. During these moments, they expanded on each other's answers or comments with their own thoughts and experiences. The prospective teachers looked at and made eye contact with each other when they spoke instead of directing their attention to the professor. They used hand movements to exaggerate their statements (Field notes 2, 4/12). In one such instance, Mike asked Amy a question as to why she did not grade homework. His question to her sparked a discussion as she shared her perspective on homework.

Mike [to professor]: May I ask her a question?

Mike [to Amy]: Why don't you grade your homework?

> Amy: Because I don't necessarily expect them to get them all right. I hope they would get it correct, but my goal for homework is a little extra practice if you're struggling with something and usually I give homework assignments where the answers are given in the back of the book so they can check. In my opinion, the goal is for them to work on it and practice so when they come back to class they have questions and another class is dedicated to them asking questions about something they don't understand. So my point is I want them to at least try it, at least work on it. But that's just me.
>
> Prospective teacher 1: How can you tell that they actually attempted to do the questions as opposed to copy down some work and hope it looks like they did it?
>
> Amy: Right, and that goes back to homework is only a small percentage of the grade and if that's what you want to do then that's what you want to do. You know what I mean? That's your choice. We're in high school now. That's just how I feel. If you don't want to do it, then that's you, but this is to benefit you and I think it's really important (Audio recording transcription 2, 4/12).

These instances, however, remained limited. The prospective teachers sought confirmation of their responses and comments from the professor with the direction of their comments toward her and the nature of their questions such as "What do you think is a good essential question?" and "What is your opinion on students helping create rules?" Therefore, their discourse continued to be aimed at the professor and not to each other (Audio recording transcription 1, 3/26; Audio recording transcription 2, 4/12; Field notes 1, 3/26; Field notes 2, 4/12).

The Prominence of Questioning in Directing and Sustaining Discourse

Researchers have noted the prominent role questioning plays in classroom discourse (Cobb et al., 1997; Wood et al., 1993). Pierson (2008) distinguished questioning as the primary means of instruction. The use of questioning can enhance or limit discussions based on the cognitive level of the questioning (Krathwohl, 2002; van Zee & Minstrell, 1997). High-level questioning provokes high-level thinking and application of knowledge (Hiebert & Weane, 1993). In this study, questioning played a significant role in directing and sustaining discourse during both the mathematical tasks and pedagogical discussions. The data demonstrated the different roles questioning assumed depending upon the subject of the discourse. The data depicted questioning as an assessment tool during the mathematical tasks as the professor assessed the mathematical knowledge and understanding of the prospective teachers. During the pedagogical discussions, questioning assumed a different role as it was used as an impetus for conversation on pedagogical issues.

Questioning as a Form of Assessment of Mathematical Knowledge and Understanding

Questioning constituted the major function of the classroom discourse during the mathematical tasks. It comprised 52% of the total verbal statements made by the professor. Of the professor’s 151 verbal statements during the mathematical discourse, 78 were questions. The questions she asked provided evidence of the prospective teachers’ mathematical knowledge. Her questioning spanned various levels of thinking ranging from lower levels of cognitive ability such as recall and comprehension questions to higher levels of cognitive ability such as application and justification questions. For the purpose of this study, I categorized low-level cognitive questions to include questions that required recall of information and questions that asked for understanding of concepts. High-level cognitive questions were categorized as questions that asked for application, analysis, and evaluation of knowledge. This categorization was based on the taxonomy of cognitive processes developed by Anderson et al. (2001).

The professor asked 50 (64%) low-level, recall or comprehension questions during the three mathematical tasks. Based on Anderson et al.'s (2001) taxonomy, these questions were *remember* and *understand* questions. These questions required the prospective teachers to provide prior knowledge or answers that were from rote memory. An example of a low-level cognitive question asked by the professor was "How do I find the mean for a set of data?" and "How many degrees are in an equilateral triangle?" (Audio recording transcription 1, 3/26). The professor asked 28 (36%) high-level, explanatory, application or synthesis questions that required the prospective teachers to process information and apply their knowledge to different situations. These questions included *apply*, *synthesize*, and *evaluate* questions. An example of a high-level question was "What interpretations of the data can you make based on the box and whisker plot?" (Audio recording transcription 1, 3/26).

The prospective teachers also participated in questioning during the mathematical tasks. The prospective teachers asked 16 low-level questions that were primarily clarification questions to an answer or comment or to gain understanding of a concept. An example of the prospective teachers' questions to the professor included "How do I enter the equation into the graphing calculator?" (Audio recording transcription 2, 4/12). A prospective teacher asked one high-level question to the professor in which she inquired about any activities that effectively show how to prove the definition of a radian.

Their mathematical questions were directed toward the professor and not each other. Table 8 shows the type of questions asked by both the professor and the prospective teachers during the mathematical tasks.

Table 8

The frequency of questions asked during mathematical tasks

	Low-Level Questions	High-Level Questions
Professor	50	28
Prospective teachers	16	1

As affirmed by the professor through her member checking of the findings, questioning for the professor served the purpose of assessment of mathematical knowledge and understanding (Member checking email, 10/1). The questions she asked demonstrated to her the prospective teachers' extent and depth of knowledge of mathematical concepts when they were immersed in a mathematical task. Her questions were often specific with a single, correct response. Table 9 reflects the level of questions asked by the professor and the prospective teachers.

Table 9

Frequency of the level of questioning during mathematical tasks

	Remember	Understand	Apply	Analyze	Evaluate	Create
Professor	21	11	10	5	3	
Prospective teachers	4	12	1			

The professor's questioning allowed her to formatively assess the prospective teachers' procedural understanding of mathematical concepts. For example, the professor assessed the prospective teachers' knowledge of triangles through her questioning as she led them through the derivation of the unit circle (Field notes 1, 3/26). Her questioning assessed their knowledge of the angles of equilateral and isosceles triangles through questions such as "What is the measure of the angles of an equilateral triangle?" or "If I drop a perpendicular here, what is the measure of this angle?" (Audio recording transcription 1, 3/26). This questioning, however, promoted recall knowledge. At the end of the mathematical tasks, high-level, application questions appeared.

After the prospective teachers created their box and whisker plots during the data analysis task, the professor asked them questions such as "What can you conclude about the data based on the graph you created?" and "What connections can be made between the data and the graph?" (Audio recording transcription 1, 3/26). This questioning asked the prospective teachers to apply their knowledge and make conjectures about the data they had collected. For the polynomial task, the participants' questioning did not align with the questions within the task. Within the task itself, nine questions asked for explanations of reasoning, making connections, and describing relationships (Document 5, 4/12). However, during the discourse about the polynomial task, four questions asked for explanations, connections or relationships and aligned with the task's questions.

The prospective teachers' knowledge *about* mathematics was missing from the questioning during the mathematical tasks and the related discourse. They participated in the *what* of completing the task but did not address the *how* of effectively teaching the task to students. The professor did provide the goals of the tasks to the prospective teachers both verbally and in written form. She provided them with the state performance standards that aligned with the mathematical task (Field notes 1, 3/26; Field notes 2, 4/12; Document 3, 3/26; Document 4, 3/26; Document 5, 4/12). An example of the professor's verbal acknowledgement of the task goal occurred at the beginning of the polynomial task. She began the task by stating, "The focus is on having students make connections on different classes of polynomials. So we are going to be looking at that relationship, that connection" (Audio recording transcription 2, 4/12).

For each of the three mathematical tasks, very few questions occurred about the surrounding issues of *how* to use the task with middle and high school students. Examples of these type of *how* questions include how to discern prior knowledge, how to address misconceptions, how to provide multiple representations of the mathematical concept, how to offer alternate explanations, or how to formatively assess student learning (Ball, 1990; Hill et al., 2004; Hill et al., 2008; Graeber, 1999; Shulman, 1986). Ball (1990) described the information about the teaching of mathematical concepts as knowledge *about* mathematics. Hill et al. (2008) described knowing how to teach mathematics as knowledge of content and teaching.

During the data analysis task, the professor did ask the prospective teachers what type of prior knowledge a middle school student would need prior to beginning the activity (Audio recording transcription 1, 3/26; Field notes 1, 3/26). Yet, the prospective teachers' responses were limited in their description. The majority of questions asked by both the professor and prospective teachers during the mathematical tasks predominantly addressed mathematical content knowledge and more specifically, surface level understanding of mathematical concepts.

Each microteaching teacher asked many questions during her simulated lesson. Like the mathematical tasks, questioning played the dominant role in shaping and directing the discourse that occurred in the microteaching lessons as demonstrated by the data in Table 10.

Table 10

Frequency of questions asked during microteaching lessons

	Low-Level Questions	High-Level Questions
Teachers	59	12
Students	5	3

Note. Teachers represent the three prospective teachers who presented a lesson and students represent the prospective teachers who assumed the role of students during the lesson.

The type of questions Jill, Lisa, and Amy asked showed that questioning served the purpose of assessment (Field notes 3, 4/26; Audio recording transcription 3, 4/26). The questions they asked were primarily low-level questions that solicited specific answers. Through their questioning, Jill, Lisa, and Amy judged the prospective teachers' comprehension of the mathematical task. Amy's questioning during her Tic-Tac-Know activity showed her use of questioning as assessment (Document 8, 4/26). After the class had completed the activity, Amy brought them together to discuss their answers.

> Amy: All right, so let's bring it back together. I hope you guys finished. Let's just go over it real quick as a class. What did you get for the very first box?
>
> Prospective teacher 1: 1 and 9.

Amy: 1 and 9. Did everyone get 1 and 9?

Prospective teachers: Yes.

Amy: All right. Can we go to the right? What did you get?

Prospective teacher 2: -8 and -2.

Amy: Is that right?

Prospective teachers: Yes (Audio recording transcription 3, 4/26).

This question-and-answer format continued for all of the boxes within the task. Amy did not ask for any explanations for the responses nor did she ask anyone to demonstrate how they arrived at their solution. She affirmed their answers with the nod of her head and moved on to the next box (Field notes 3, 4/26). At the end of her questioning, Amy asked what was discovered from the task.

Amy: What did we discover?

Prospective teacher 1: That we can find two numbers when added together come up with a common product.

Prospective teacher 2: That for each set of numbers, there is one solution.

Amy: Right, you can't change the solution. Very good. Anything else that was noticed? No? Yes? All right, so basically what we see is that we have two numbers that we multiply together to give you a number added together. That's all we saw, all right? (Audio recording transcription 3, 4/26).

The prospective teachers' mathematical discourse during the simulated lessons was dominated by low-level, recall questions. These questions represented 83% of all the questioning asked by Jill, Lisa, and Amy. There lacked a concerted focus on higher-level cognitive thinking evidenced by the small number of higher-level questions asked during the simulated lessons. Higher-level cognitive questions represented 17% of the questions asked by the three prospective teachers. Table 11 reflects the type of questioning Jill, Lisa, and Amy used during the simulated lessons.

Table 11

Frequency of the levels of questioning during microteaching lessons

	Remember	Understand	Apply	Analyze	Evaluate	Create
Teachers	45	14	8	4		
Prospective Teachers	3	2	3	2		

Note. Teachers represent the three prospective teachers who presented a lesson and students represent the prospective teachers who assumed the role of students during the lesson.

Instances occurred where Lisa, Jill, or Amy failed to recognize the opportunity for higher-order thinking from both the questions they asked and the questions asked of them. During Jill's lesson on theoretical probability, she asked the question, "Why do you think we want to do this today?" Amy answered with "Because we're going to the mall." Jill's response to Amy was, "Because we're going to the mall, no. Because what's the unit on?" (Audio recording transcription 3, 4/26). Jill's response of "Because what's the unit on?" signified a lack of mathematical purpose to the task. Learning took place because it had to, because it was part of a unit in a curriculum.

The manner in which Jill responded to Amy's response reappeared later in the lesson. In the following interaction, Jill's response limited another opportunity to promote engagement through a constructive mathematical conversation. When a prospective teacher asked Jill what to do if each outcome of flipping a coin was not equally likely, Jill's response signified a lack of mathematical purpose. Jill responded, "If it's not equally likely, then we're going to have to do a different task and we're not doing that this year. That's ninth grade math. This is eighth grade math" (Audio recording transcription 3, 4/26). Her response halted any higher connections that had the potential to occur for the prospective teacher. According to Jill's response, if question did not pertain to eighth grade mathematics, then they did not explore an answer. She established boundaries for the learning within the task.

Questioning as Prompts for Perspectives During Pedagogical Discourse

Questioning during the pedagogical discussions provoked discourse on a variety of pedagogical topics. The majority of the professor's questioning consisted of open-ended questions that asked for the prospective teachers' opinion along with their reasoning and application of current pedagogical knowledge. The professor's goal was to prompt thought, reflection and debate about pedagogical issues (Member checking email, 10/1). The type of questions the professor asked reflected her goal. Her questions often began with "What do you think" or "Why do you think?" (Audio recording transcription 1, 3/26; Audio recording transcription 2, 4/12). The prospective teachers engaged in more contributory communication with the professor because they shared their thoughts and perceptions on pedagogical issues. For example, the professor asked the class, "Do you think mathematics is complicated?" (Audio recording transcription 2, 4/12). This question solicited the opinion of the prospective teachers. Many of the questions also asked them to respond within the context of their field experience. For example, at the beginning of a discussion regarding discipline, the professor prompted the class by asking, “What kind of discipline are you seeing in your classrooms?” (Audio recording transcription 1, 3/26). Jill situated her response to the question in the context of how her cooperating teacher handled discipline. When the professor sparked a discussion regarding mathematical review for standardized tests in classrooms, she began by asking the class, “What kind of review are you doing with your students?” (Audio recording transcription 1, 3/26). Ann’s response described the methods her cooperating teacher utilized to prepare her students for the upcoming state test. Questioning, while still key, did not play a pivotal role in the shaping of the discourse during pedagogical discussions, as evidenced by the data in Table 12.

Table 12

Frequency of questions asked during pedagogical discussions

	Low-Level Questions	High-Level Questions
Professor	21	28
Prospective teachers	11	9

The data depicted a higher frequency in the number of high-level questions that occurred during the pedagogical discussions. The professor asked 21 (43%) low-level questions that were yes or no questions or questions that asked for an opinion without an explanation such as “Have you seen demonstration being used in a classroom?” (Audio recording transcription 2, 4/12). The professor asked the prospective teachers 28 (57%) high-level, explanatory questions such as “As you reflect on the article, what are some points that you walked away thinking, ‘Ok, I have a better idea of that now?’” (Audio recording transcription 1, 3/26). She asked the prospective teachers to explain their reasoning, opinion, or to justify their thinking (Audio recording transcription 1, 3/26). Table 13 reflects the participants' level of questioning that occurred during the pedagogical discussions.

Table 13

Frequency of the level of questioning during pedagogical discussions

	Remember	Understand	Apply	Analyze	Evaluate	Create
Professor	12	9	8	4	6	
Prospective teachers	8	3	6		3	

The prospective teachers asked nine (45%) high-level questions during the pedagogical discussions. They directed two of the nine questions at other prospective teachers rather than the professor. The 11 (55%) remaining questions they asked were clarification questions about a statement or question that were directed toward the professor. Additionally, the prospective teachers' questions sought the professor's opinion about a specific topic. For example, the class engaged in a lively discussion over what characteristics constituted an effective essential question. A prospective teacher asked the professor, "In Wednesday's class, we talked about the essential question and her [the professor's] big thing about it was that the essential question has to be an open-ended question. It's not allowed to have a yes or no, correct or incorrect answer. How do you feel about that statement?" (Audio recording transcription 1, 3/26). This question, which sought the professor's opinion, sparked a discussion on open-ended questions and their applicability in mathematics.

The Use of Scaffolding to Deliver Mathematical and Pedagogical Knowledge

Williams and Baxter (1996) described analytical scaffolding as imparting information about a concept in order for an individual to form an understanding, deepen the level of current understanding or to develop knowledge. The use of scaffolding can enhance learning when used purposefully, but can also be limiting to learning when used excessively. In this study, scaffolding was used to deliver both mathematical and pedagogical knowledge. During mathematical discourse, the scaffolding provided by the professor limited the prospective teachers' explicit ownership of learning. She provided the majority of the mathematical statements. During pedagogical discourse, the professor and prospective teachers provided scaffolding to supplement the prospective teachers' current knowledge of pedagogical issues.

The Professor's Expansion of Prospective Teachers' Mathematical Responses

Analytical scaffolding appeared as a contributing factor that shaped the classroom discourse. According to William and Baxter (1996), analytical scaffolding is the process in which a teacher provides information to students in order to guide them to a desired understanding of a concept. They described it as the way a teacher guides a student, through purposeful questioning or giving key pieces of information, to learning that was previously unknown or needs reinforcement.

The professor excelled at providing analytical scaffolding to the prospective teachers. In fact, the professor provided a great amount of analytical scaffolding to the prospective teachers whether they mathematically required it or not (Field notes 1, 3/26; Field notes 2, 4/12; Field notes 3, 4/26). The sheer number of explanations she provided evidenced her use of scaffolding. During the three mathematical tasks, the professor provided 56 mathematical explanations, which comprised of 37% of her total mathematical verbal statements. The scaffolding also provided the professor a way of continuing the dialogue about a mathematical concept. The prospective teachers also provided analytical scaffolding. Scaffolding represented 19% of their mathematical statements. Their scaffolding occurred when the professor probed for an explanation or justification to a response.

> Professor: All right, what about the relationship between the y-intercept and the lines? Are you able to see any relationship there?
>
> Prospective teacher 1: It's the product of the two y-intercepts.
>
> Professor: Yes, yes, you're right. Okay, say that again.
>
> Prospective teacher 1: It's the product of the two y-intercepts of your lines because your x's are going to be zero so the only thing that will be left is the product of your two intercepts.
>
> Professor: Do you guys see that? Excellent (Audio recording transcription 2, 4/12).

By probing for a restatement of her answer, the professor encouraged an explanation from the prospective teacher. Her second response provided a justification to her answer. She provided analytical scaffolding to the rest of the prospective teachers through her justification of thought.

The professor's scaffolding served two purposes. The professor verified the purposes of the scaffolding through member checking (Member checking email, 10/1). The first purpose of the analytical scaffolding was to provide mathematical information to the prospective teachers so that they could complete their task. The data analysis task presented such an example. After discussing the selection of the bar graph to display the data, the professor provided scaffolding to ensure that the prospective teachers could create a bar graph.

> Professor: Do the bars touch though? With the bar graph, the bars don't necessarily touch.
>
> Prospective Teacher 1: She's doing each person, each person is two bars.
>
> Professor: Yeah, those two touch, but then I think you'd have to leave some space.
>
> Prospective Teacher 1: So there are seven groups of two bars.
>
> Professor: Let's do it. Let's practice because didn't I give you graph paper? Let's practice a bar graph. So a bar graph is, you're going to have your majors here and then you're going to have your frequency down here, is that right? You agree? (Audio recording transcription 1, 3/26).

The professor provided necessary information for the creation of the bar graph such as whether the bars touched and which labels should be located on the two axes. While this was information that the majority of the class might have already known, the professor decided that it needed to be provided to the entire class in the event that someone did not know. Another example occurred while the prospective teachers completed the unit circle task. The purpose of the task was to derive the sine graph based on the unit circle. The prospective teachers used spaghetti to measure the distance of points from the point (0,1). They then placed the pieces of spaghetti along the x-axis on a coordinate plane. The placement of the spaghetti pieces form the sine graph. The professor led them through the steps of creating the unit circle.

> See, what you're going to do is you're going to go all the way around, and you're going to put a little dot there because that represents 30. And then you're going to go there and you're going to put a dot there because that's going to represent 45 when you stretch it out on the x-y. Then that's going to be 60, that's going to be 90…Except these are going to be your thetas down here. You with me? And isn't this your like numbers, that would be 0, 1, 2, and so you would just take it, if that's 30 degrees and you would just glue your spaghetti there. Measure from there to there because you want to transfer that y value because that's what's going to give you your sine graph. Y=sine theta. And so what this does, it allows your students to see that the unit circle; you can get two graphs out of that unit circle. If you're looking at your y values for sine, you're going to get the sine function if you're taking that distance. How would you get your cosine? You get the x distance (Audio recording transcription 1, 3/26).

Instead of the prospective teachers exploring the task on their own, the professor provided them with explicit directions to create the graphs.

The second purpose was to reiterate important conceptual information. This type of scaffolding occurred during the data analysis task after a prospective teacher had given a response for the skewness of the box and whisker plot created by the data.

> Professor: All right, for your first piece of data you found the mean, correct? And you found the median. Now, if we were talking about the distribution of the data in terms of skewness and symmetry, what can we conclude about the first piece of data, or group of data, in terms of, is it skewed or is it symmetric? I know if I

have a curve such as this, I have symmetry in my data, correct? What does that mean for the mean and the median?

Prospective teacher 1: They would line up.

Professor: They'd be the same, wouldn't they? Your mean and your median would be the very same thing. How close are we with that first piece?

Prospective teacher 2: Not very. The median is bigger.

Professor: So the mean is less? What kind of skewness is that? So here's my, you said that my mean was less than my median? My median was here perhaps? [points to a specific point on the graph] so my skewness is going to be like that perhaps? So negative skewness. What about the other one?

Prospective teacher 2: It's closer.

Professor: It's a little closer so you might have a little more symmetry there, but it's not perfect though. Is one bigger than the other?

Prospective teacher 1: The mean was bigger.

Professor: The mean was bigger? So you're going to have a positive skewness there. You did better with that group of data (Audio recording transcription 1, 3/26).

Through the professor's dialogue and questioning with the prospective teachers, she restated the connection of a graph's skewness to the distribution of data. In doing so, the prospective teachers had the opportunity to make connections about the relationship between data and their graph. Those connections had the potential to enable the prospective teachers to apply their knowledge at a cognitively higher level. However, the prospective teachers did not discuss their connections; therefore, they remained unknown.

The completion of the polynomial task provided another example of the scaffolding's purpose as the professor commented on the importance of being able to work backwards.

> When you are working backwards, there's something else that I want you to notice too so I want you to go back to that screen where you have all your graphs on that screen, you know all three of them. And if you kind of divide your function up, you might have something that looks like this. You know, something that looks kind of like that. And then this is your x, y coordinate. So what I want you to notice is that if you look at this part of the graph, do you see how this part of the graph of the parabola is above the x-axis so that would indicate that this is positive, correct? Well, if you look here and you take the product of these two because these are below the graph, these are negative, right, values. So when you multiply a negative, and you take that positive, a negative times a negative, that part of the parabola should be above the graph. Does that make sense to everyone? So if you section each part of that off, you should be able to determine when something is positive or negative. My drawing is a little off here, but if you look at your graph you should be able to tell that. Now that's going to help you when you're trying to work backwards, like you're given a parabola and you want to try to find the two linear functions that are going to give you that parabola. So you'll need to keep that in mind (Audio recording transcription 2, 4/12).

Within the mathematics methods course, the professor provided the prospective teachers with ample analytical scaffolding even though she stressed the importance of mathematical communication as a way to make connections. During a conversation, the professor commented on the importance of promoting student communication. She stated, “You want to make sure that you are giving your students an opportunity to communicate the mathematics to each other because I think that’s the best way that you can help your students understand, or help you understand what they understand” (Audio recording transcription 2, 4/12). Yet, the prospective teachers were not challenged to communicate with each other in their own discovery of learning as evidenced in the lack of probing, explanatory questions asked during mathematical discourse (Field notes 1, 3/26; Field notes 2, 4/12; Field notes 3, 4/26).

Pedagogical Knowledge Transmitted Through Discourse

Even though the focus was pedagogy, the professor and prospective teachers engaged in discourse in which analytical scaffolding, in terms of pedagogical knowledge, remained an important element. During a discussion in which they discussed a reading on student learning, the professor asked the entire class a question about guided notes.

> Professor: Do you remember why she suggested that it might be a good idea to do guided notes?
>
> Prospective teacher: Create more time.
>
> Professor: You have more time. Yeah. Not only that, but for the students what does it do for them?
>
> Prospective teacher: Relieves the stress.
>
> Prospective teacher: Allows them to pay attention while they’re reading over it.
>
> Professor: And you’re able to delve perhaps more deeply into the content if you’re not waiting for them to write down the notes before you can move on (Audio recording transcription 2, 4/12).

In this conversation, the professor provided the prospective teachers scaffolding in order for them to capture the pedagogical benefit of guided notes. While the prospective teachers' responses advocated the use of guided notes, the professor redirected her questioning to specify the benefits for students rather than for the teacher. She challenged them to think more deeply and globally about using guided notes. Additionally, the professor summarized the major benefit of guided notes as a tool that provides deeper focus of content when students still do not reach a high level of understanding.

Scaffolding for pedagogy included the professor providing research evidence on a particular practice or idea. As the class participated in discussions on homework and discipline, the professor provided validation or refutation of ideas. As the class discussed the nature and frequency of review in the classroom, the professor provided research about review with a specific group of learners.

> And the other thing also, the research, I've been looking at some research that says that the best, and this is with ESOL learners too, the best thing to do for students who are deficient is not to start the beginning of the year reviewing, like you want to spend two days reviewing what they should have learned in sixth and seventh grade if you teach eighth grade. The research says that the best thing to do is to review with the student at the time that you are covering that particular topic (Audio recording transcription 1, 3/26).

The professor expanded through scaffolding, the prospective teachers' strategies for teaching. Throughout their discourse, the professor offered the prospective teachers a variety of instructional strategies (Field notes 1, 3/26; Field notes 2, 4/12). One such instance occurred during the discussion in which the professor described the various modes of instruction. Discussion was one such mode. As she described what a discussion should look like, the professor provided a technique to vary student discussion.

> You can have whole group discussion or you can put your students in groups and provide a prompt. Or you can do a jigsaw type task where you break them apart and you put A, B, C, D, E each in a group to become the expert, they become the expert. You go back to the group, or you put all your A's together and all your B's together and all your C's together and they become the expert when you come back together then you have A, B, C all together and you can explain what you learned as an expert (Audio recording transcription 2, 4/12).

Not only did the professor provide a description and restatement of a discussion, she also provided a specific strategy to promote student discussion. She provided the prospective teachers with pedagogical scaffolding. The professor provided more scaffolding than perhaps the prospective teachers required. In several instances, the professor had the opportunity to encourage them to provide an explanation rather than providing it herself. This occurred during a discussion on the modes of instruction. As the professor introduced the different modes of instruction, she elaborated on the characteristics of each mode.

> Professor: All right so there's lecture. You know what lecture is. They sit, they take notes, and then you say, "Are there any questions?" Do you know it when you see it? Is it student centered?
>
> Prospective teacher: No.

Professor: Absolutely not, if that's all you're doing. Ok, so the teacher talks, and perhaps with some use of the chalkboard, and the students are just going to be listening quietly. Sometimes they take notes…

Professor: Question and Answer. You ask a question, the student will answer, there's a reaction to it. You'll ask another question either of the same student or another student….They could be taking notes as well. Have you seen it in your classrooms? (Audio recording transcription 2, 4/12).

The professor had the opportunity to promote processing and reflection by asking the prospective teachers to offer their own definitions and descriptions of each mode rather than providing it for them. Their body language signaled that they possessed prior knowledge on many of the modes of instruction the professor described. They appeared aware of each mode of instruction through their reactions and body language during the discussion such as their nodding of heads (Field notes 2, 4/12). The professor also asked them if they knew each mode as they began discussing them. They affirmed that they had. The professor actively engaged in the pedagogical discussions and therefore, provided more scaffolding than the prospective teachers may have required.

The Influence of Engagement in Shaping Discourse

The data depicted prospective teachers who demonstrated varying levels of engagement and participation during classroom discourse. Their engagement and participation level varied depending on the topic of discourse. During mathematical discourse, the prospective teachers displayed engagement levels that were compliant but unenthusiastic. They participated in the discourse but did not sustain the discourse on their own. During pedagogical discourse, the prospective teachers expressed a higher level of engagement that they reflected through their body language and participation. Interviews identified pedagogical knowledge as the prospective teachers' expectations for learning rather than mathematical knowledge.

The Lack of Engagement in Mathematical Discourse

The classroom observations, interview responses, and writing prompts reflected that the prospective teachers were not as eager to engage in discourse about mathematical tasks as they were in pedagogical discussions. They demonstrated their lack of engagement during the mathematical tasks through their limited conversations regarding the tasks and in their body language such as their heads turned down (Field notes 1, 3/26 ; Field notes 2, 4/12; Field notes 3, 4/26). Some of the prospective teachers worked on their computer during the mathematical discourse (Field notes 1, 3/26).

When the interview participants commented about the lack of engagement toward the mathematical tasks, they each attributed it to various reasons. Amy commented her lack of engagement was due to her dislike of the learning tasks. She commented, "I don't particularly like learning tasks, but I know that my professor does, therefore I didn't what to say too much about it" (Interview 1, 9/16). Ann attributed it to her concern of pedagogy and operational issues rather than mathematics. She stated, "I think this is true because each of us in this course has a common concern about our own classroom management styles, and wanted to bounce ideas off of each other. I think that we are all pretty comfortable with our mathematics abilities, but need some reassuring and ideas in the area of classroom management and other areas of teaching" (Interview 1, 9/15). Beth stated that she could not attribute the lack of engagement to one particular reason (Interview 1, 9/23).

The prospective teachers, who had degrees in mathematics or related fields, displayed a strong confidence in their mathematical ability (Field notes 1, 3/26; Field notes 2, 4/12). The interview responses expressed the three prospective teachers' mathematical confidence. Graduate Record Examination (GRE) scores of four of the participants supported this high mathematical confidence. Of the four scores I was able to view, three of the four scores were above 700, which signified strong mathematical ability (Document 9, 10/1). Ann responded to the class's high mathematical confidence stating, "I think we are all pretty good at math. We all know our stuff. Maybe some of us didn't know something specific throughout the activities but all in all, I think everyone has a pretty good handle on the math part" (Interview 1, 9/15). Amy acknowledged her high mathematical ability stating, "I feel I am strong conceptually in mathematics. I think I know more than just how to solve it. I know why I can solve it the way I do. It is important to know more than just procedures. I try to convey that to my students. To understand the why in math and not just how to do it" (Interview 1, 9/16). Beth affirmed her mathematical confidence, but also acknowledged the need to expand her knowledge. She commented, "I feel that I am still growing in my knowledge of the concepts behind the mathematics that I have always been taught. I feel that I will continue to grow and learn and will always be my own student as well" (Interview 1, 9/23).

The prospective teachers expressed their confidence through their responses to the writing prompts completed after the class. In the writing prompts, 11 of the 13 total responses stated that they had shared mathematical ideas with the class. While five of the responses also stated that they might have viewed a mathematical concept in a new way, the respondents never responded that they learned a new mathematical idea through their discourse. One respondent's justification to his or her selection stated, "Our discussion on the data analysis task jogged my memory on box and whiskers plots which I hadn't studied in a long time. Once I saw it though, I remembered it. My memory is jogged often but I don't think I am ever learning anything new really" (Writing prompt 1, 3/26). In response to the writing prompt after the class in which the unit circle task was completed, one respondent had selected that he or she had viewed mathematics in a new way, remarking that "Deriving the unit circle was interesting to see and know but it is not something that I would ever do or show to my students because they would never get it" (Writing prompt 1, 3/26). The discourse did not provoke reflection or change in his or her thinking based on their writing prompt responses. The writing prompts, which served the purpose of promoting reflection on the discourse, demonstrated that the prospective teachers stated they were participants in the classroom discussion, but were not changed by it.

During the mathematical discourse about the unit circle and the derivation of the sine graph, one of the prospective teachers, Mike, explicitly expressed his mathematical confidence. The professor led the prospective teachers through questions as they derived the degrees of the unit circle. The professor then gave instructions on what they would do next in order to move from the unit circle to creating the sine graph. She questioned Mike's understanding of the process.

> Professor: Are you okay because you're not, I don't feel like you're with me and you need to know.

Mike: I'm fine. It's just that I kind of already know the answers to everything.

Professor: But your students don't, do they?

Mike: This is new and exciting, exploratory?

Professor: I'm trying to help your students discover, that's all I'm trying to do.

Mike: The exercise is new to me, as far as that goes (Audio recording transcription 1, 3/26).

Mike was not eager and engaged in the task because he felt that he was competent in the mathematical concepts connected to the task. Another occasion demonstrated the prospective teachers' mathematical confidence. The professor asked the class, "Do you think mathematics is complicated?" The prospective teachers collectively responded, "No." The professor repeated the question with a more questioning tone, "Do you think mathematics can be complicated?" One prospective teacher responded, "For the students." Another prospective teacher remarked, "We already know how to do math" (Audio recording transcription 2, 4/12). The prospective teachers did not see the mathematics within the tasks as being problematic.

The Engagement of Prospective Teachers During Pedagogical Discourse

The data showed that the prospective teachers were engaged in pedagogical discourse. Classroom observations showed that the prospective teachers were attentive during these discussions (Field notes 1, 3/26; Field notes 2, 4/12). They looked at the person who was talking and no one worked on other items, such as their computers, during these discussions (Field notes 1, 3/26; Field notes 2, 4/12). The data from the previous tables showed an increase in their questioning and overall verbal statements. Recall during the mathematical tasks, the students seemed complacent and disengaged in both the tasks and the discourse surrounding them. This was not the case during pedagogical discourse. The prospective teachers exhibited a willingness to engage in discourse about pedagogical issues.

Unlike mathematics, the prospective teachers were not confident in their pedagogical knowledge and skills. In her response as to why the prospective teachers were engaged in pedagogical discourse, Beth confirmed their possible lack of confidence stating, “It could be as a result of us feeling less adequate in the area of running the classroom than the mathematics” (Interview 1, 9/23). Ann also reiterated this by remarking, “Especially since we are all new to the teaching profession, we are not as confident” (Interview 1, 9/15). The prospective teachers engaged in pedagogical discourse because they felt invested in the information they could receive.

The prospective teachers engaged in discourse that centered on topics such as discipline and homework. The length and depth of the discussions and the higher frequency of participation in the discourse by the prospective teachers evidenced their interest (Field notes 1, 3/26; Field notes 2, 4/12). The time that they spent discussing these issues resulted in half of the allotted three hours of class time. They solicited each other’s ideas on how to reduce copy paper when giving guided notes. Issues such as lack of paper to make copies and managing classroom behavior were at the forefront of their discussions. The discussion on copy paper alone lasted approximately 25 minutes (Field notes 2, 4/12).

The prospective teachers’ desire was to learn specific teaching strategies and operational knowledge of teaching. The topics within the pedagogical discourse and the interview responses demonstrated their desire. I termed operational knowledge as the knowledge of not teaching a class, but running and managing a class. Ann commented on her observation of the way the class changed the focus of their discussions to operational issues. She stated, “When it came time to discuss math content, we did have a tendency to frequently change to focus to either teaching methods or operational issues” (Interview 1, 9/15). Amy also noticed the shift of focus from her peers commenting, “People wanted to talk about things happening in their schools and that is fine” (Interview 1, 9/23). Ann affirmed her desire for operational knowledge through her response about her lack of engagement in mathematical tasks. She stated her area of concern was classroom management (Interview 1, 9/15). All three interview participants described their desire to have gained teaching strategies as a result of the course. They each stated this desire as their perceived purpose of the course. Amy commented that the purpose of the course was to “find new methods of teaching mathematics to students” (Interview 1, 9/16). Beth stated, “I thought the purpose of the course would be to facilitate in developing not only our own teaching methods, but also to become familiar with any background knowledge necessary in becoming an effective mathematics teacher, such as GPS curriculum and NCTM standards” (Interview 1, 9/23).

Summary

The data presented in this chapter reflects the existence of two separate discourses on two separate entities-mathematics and pedagogy. Mathematical discourse revolved around the completion of three mathematical tasks and three microteaching lessons. Mathematical discourse was teacher-centered and vertical in the directional flow of the discussion. Mathematical conversations occurred primarily between the professor and the prospective teachers with little horizontal interactions among the prospective teachers. The professor dominated the discourse by asking questions to assess mathematical understanding and providing scaffolding of concepts in the form of mathematical statements. The prospective teachers exhibited low levels of engagement toward the mathematical tasks. The mathematical tasks did not align with their stated expectations for the methods course.

Pedagogical discourse revolved around discussions on general teacher pedagogy and operational issues. Pedagogical discussions occupied half of each class meeting. During pedagogical discourse, the prospective teachers became active participants contributing responses to the questions asked by the professor. They also asked questions of the professor and their peers. The professor continued to be a prominent figure in the discourse. She questioned the prospective teachers with open-ended questions that led to whole-group discussions. The prospective teachers engaged in the pedagogical discussions as the discourse aligned with their stated expectations toward the methods course.

CHAPTER V

CONCLUSIONS AND DISCUSSION OF FINDINGS

In this chapter, I answer my research questions and present a discussion of my findings situated in a theoretical context. Based on the data generated from the classroom discourse in a mathematics methods course, four findings on classroom discourse are discussed. One finding concerns the overall communication of the methods course and its discrepancy from the discourse promoted by reform efforts. Additionally, I discuss the role questioning plays in influencing the flow and direction of the discourse. Other findings include the factors that influenced the engagement of the prospective teachers toward discourse and finally, the forms of mathematical knowledge for teaching that the discourse facilitated and hindered opportunities for development.

Answers to the Research Questions

In Chapter one, I asked the following questions related to the present study:

1. How does discourse appear within the instructional environment of a mathematics methods course for prospective teachers?
    a. What are the patterns, functions, and nature of classroom discourse?
    b. What role does questioning play in shaping classroom discourse?
2. How did the discourse in the mathematics methods course facilitate or hinder opportunities for prospective teachers to develop mathematical knowledge for teaching?

From the findings presented in Chapter Four, I present the following answers to my research questions:

1. The discourse within a mathematics methods course varied depending upon the nature of the tasks involved and the topics of discussion. Mathematical tasks solicited discourse that was primarily uni-directional with little student-to-student discourse. Brendefur and Frykholm (2000) describe uni-directional communication as limiting to the meaningful construction of knowledge. Pedagogical discussions solicited discourse that was vertical but contributory with increased student-to-student discourse. While contributory, the discourse still lacked the reflective qualities that Brendefur and Frykholm identified as necessary for meaningful learning. Analytical scaffolding and questioning were dominant features of both types of discourse. In mathematical discourse, questioning primarily served as a means of assessing mathematical understanding of concepts. The frequency of low-level questioning facilitated procedural learning of mathematics. The professor's scaffolding of mathematical responses limited the prospective teachers' accountability of their own thinking.
2. Hill, et al. (2005) characterized mathematical knowledge for teaching as including "explaining terms and concepts to student, interpreting students' statements and solutions…[and] using representations accurately in the classroom…" (p. 373). The discourse in the mathematics methods course about the mathematical tasks did not focus on how students might interpret and approach the task, how to use the task with students to build conceptual understanding, how to adapt the task for particular learners' needs, or other such pedagogical content issues. Rather, the

discourse during the mathematical tasks pertained to procedural aspects of doing mathematics. Even the pedagogical discourse in the methods course revolved around broad, general pedagogy, or operational knowledge of classrooms, such as behavior management. Opportunities were not provided for the prospective teachers to reflect upon and discuss how the mathematical tasks they worked on in their methods course would play out in 6-12 classrooms. In other words, opportunities were not provided for the prospective teachers to develop the mathematical knowledge for teaching that would prepare them "to carry out the work of teaching mathematics" (Hill, et al., 2005, p. 373).

Discussion of Findings

This section of the chapter provides a discussion on the lessons learned from the findings of the study. The discussion supports the conclusions of my research questions as it synthesizes the data vis-à-vis a theoretical framework. First, the discourse of the mathematics methods class is characterized using the theoretical framework established by Brendefur and Frykholm (2000). The influence of questioning on the structure, pattern, and nature of discourse is discussed using Anderson et al.'s (2001) taxonomy. Next, I analyze the role of engagement within the theoretical context of self-efficacy, mathematical problematization and achievement theory. Finally, I address teacher knowledge within the literature of mathematical knowledge for teaching using Hill et al.'s (2008) framework to discuss the knowledge that was promoted by the classroom discourse along with the forms of teacher knowledge that were overlooked.

The Communication of Prospective Teachers Promoted Through Discourse

At the outset of this study, I had hypothesized that the communication level among the prospective teachers would be characterized as primarily contributory with some elements of reflective communication. Contributory communication is characterized as dialogue that consists of assistance and sharing between the teacher and students or among the students, in this case prospective teachers (Brendefur & Frykholm, 2000; Cobb et al., 1997). In reflective communication, the discourse promotes deep understanding and an examination of ideas with the purpose of making adjustments to prior knowledge (Cobb et al., 1997; Wood et al., 1993; Simon, 1995). I made the assumption that a level of contributory communication, if not reflective, would exist among the prospective teachers in the mathematics methods course because the prospective teachers in the course were adult learners, had a vested interest in the class, but were likely inexperienced in using communication as a tool for reflection and learning.

Discourse Differences

The findings of this study reveal that facilitating and participating in purposeful classroom discourse remains a marginally developed skill for the prospective teachers. It is a skill that requires learning and continued practice, reflection, and modification (Brendefur & Frykholm, 2000; Cobb et al., 2007; Nathan & Knuth, 2003; Simon, 1995). The prospective teachers did not demonstrate that they possessed skills to facilitate productive discourse when teaching a mathematical lesson. Furthermore, they did not demonstrate that they possessed the skills to participate in purposeful discourse. Distinct differences in the characteristics of mathematical and pedagogical discourse appeared that characterized the level of communication. The findings of this study identify the direction of the discourse as a difference between the two discourses. Characteristics between both types of discourse that differentiate their communication include the role of the professor, the direction of verbal interactions within the discourse and the role of questioning and its influence on the discourse.

Mathematical discourse. The overall communication that occurred within the mathematics methods class during mathematical discourse can be categorized as *uni-directional*. According to Brendefur and Frykholm (2000) uni-directional communication limits student construction of knowledge. Interactions in uni-directional communication do not provide an impetus for conversation. The prospective teachers' mathematical discourse did not demonstrate that they increased their mathematical knowledge through their participation in the mathematical tasks. The vertical pattern of their discourse constricted the opportunities for the *collective* sharing of ideas and perspectives. The findings of this study indicate that the environment and the discourse of the methods course typified the environment of a traditional mathematics classroom lacking the social construction of knowledge. Simon (1995) has asserted that discourse is a tool that allows learners to make connections and form mathematical knowledge as a community. Cobb et al. (1997) has described a socially-constructing environment as producing "taken as shared" knowledge (p. 258).

The prospective teachers in this study did not see the benefits of talking to each other mathematically, as evidenced by their lack of participation in mathematical discourse. Their lack of participation contradicts the stated benefits of discourse established by Ann, Lisa, and Beth in their participant interviews. Ann, Amy, and Beth stated that their expectation was to learn teaching methods, not mathematical content (Interview 1, 9/15; Interview 1, 9/16; Interview 1, 9/25). The prospective teachers did not realize that their participation *in* mathematical discourse served as a tool to teach mathematics. Characteristics of purposeful discourse and communication have to be purposefully established within the classroom as an expectation, and as a norm for learning mathematics as a community (Williams & Baxter, 1996). Otherwise, purposeful discourse is difficult to foster.

The communication level of mathematical discourse suggests that mathematical discourse should be an explicit focus of instruction. The findings of this study show that the discourse about a mathematical task occurred at a lower cognitive level than the goal, standards, and questions presented within the task. An example of this disparity occurred during the unit circle task. The goal of the task was to make connections between the unit circle and the graphs of the sine and cosine trigonometric functions (Document, 4, 3/26). The task also asked for mathematical observations as the unit circle was being created. However, during the discourse about the task, the prospective teachers did not make any connections between the unit circle and sine and cosine graphs (Field notes 1, 3/26). They also did not note any mathematical observations during the task. As time ran out, the professor had summarized the task to the prospective students.

> Ok, let’s clean up. I think everybody, do you understand this concept and the connections you can make here? Helping your students to derive it, and then from that derivation they can remember, instead of just giving it to them has more meaning. And all of your students by the time they’re in Trig, they know what an

equilateral triangle is and what a right isosceles triangle is so that certainly shouldn't be a problem (Audio recording transcription 1, 3/26).

The discourse did not challenge the prospective teachers to talk mathematically with each other to make conjectures, connections, and reach conclusions. The professor provided many of the connections for the prospective teachers as she led them through the tasks. Hiebert et al. (1996) have argued that too much information could undermine students' inquiries and prevent students from problematizing the mathematics. By making mathematical discourse an explicit focus of instruction, prospective teachers can learn to make the tasks secondary and the communication about the mathematics within the task the primary focus.

Pedagogical discourse. The prospective teachers' communication increased during pedagogical discourse to become *contributory*. Brendefur and Frykholm (2000) classified contributory communication as one's sharing of thinking contributes to mathematical conversations but the conversations lack the depth and richness to alter student thinking. Participants in contributory communication lack the skills to use discourse as a reflection tool toward their current knowledge base (Cobb et al., 1997).

It is important to note that the term pedagogy is used broadly. As defined in Chapter one, pedagogy is the knowledge, including the skills and strategies, of teaching (Shulman, 1986). The pedagogical issues that the prospective teachers actively discussed were not mathematics-specific nor were they situated in a mathematical context. I use the term *operational* to characterize many of the issues discussed during the class meetings under the umbrella of pedagogy. Because issues such as homework and classroom management are a part of teaching, they are included as pedagogical issues. Amy noticed their eagerness to engage in discourse stating, “People wanted to talk about things happening in their schools and that is fine” (Interview 1, 9/16/2010). The prospective teachers’ increased participation was evidenced by their increase in frequency of verbal interactions (Audio recording transcription 1, 3/26; Audio recording transcription 2, 4/12; Audio recording transcription 3, 4/26; Field notes 1, 3/26; Field notes 2, 4/12; Field notes 3, 4/26).

During pedagogical discourse, vertical interactions between professor and the prospective teachers still dominated the discourse, but the prospective teachers had exhibited a willingness to talk *to* each other. The findings of this study indicate that when discourse was pedagogically focused, the prospective teachers assumed a more active role in the discourse. Their increased participation signifies an increased interest in the discourse.

During the pedagogical discourse, the prospective teachers made 155 verbal statements as compared to the 123 statements made by the professor. Their participation consisted of their contribution of ideas and questions asked of the professor and to each other. They made eye contact with each other and they had positioned themselves in the direction of each other rather than the professor (Field notes 1, 3/26; Field notes 2, 4/12). The change in their body language signifies that they regard each other’s statements about pedagogy as valuable and worthy. All the prospective teachers’ signals indicated a move from uni-directional communication to contributory communication because the prospective teachers had taken an active role in the discourse by sharing their own thinking and contributing to the comments made by their peers (Brendefur & Frykholm, 2000).

The Role of the Professor

Lampert (1990) described the traditional mathematics classroom as one in which the teacher is the dominant presence. This study has revealed that the degree of the professor's prominence varied depending on the nature of the discourse. If the discourse was mathematical, she assumed a greater role through her questions, and mathematical scaffolding of concepts.

In this study, the professor made 151 mathematical statements during the mathematical discourse as compared to the 114 mathematical statements made by seven prospective teachers (Audio recording transcription 1, 3/26; Audio recording transcription 2, 4/12; Field notes 1, 3/26; Field notes 2, 4/12). The importance of the frequency of their statements is that the 151 statements made by the professor represent one individual while the 114 statements made by the prospective teachers represents seven individuals.

The findings of this study indicate that prospective teachers influence the professor's role in the discourse and in the classroom by their active participation or lack thereof in the discourse. The professor's role became one of disseminator and assessor of information during mathematical discourse, which is characteristic of uni-directional communication (Brendefur & Frykholm, 2000; Lampert, 1990; Nathan & Knuth, 2003; Williams & Baxter, 1996). However, while the professor continued to impart information to the prospective teachers during pedagogical discourse, they also participated in this dissemination as they gave each other ideas to use within their field experience. Furthermore, the professor did not assess the responses of the prospective teachers during the pedagogical discourse. Rather, her questions had asked for their ideas.

Prospective teachers' views of authority. Mewborn (1999) has found that prospective teachers looked to their teacher educator and collaborating teacher as sources of authority when they did not possess ownership of learning. This view is consistent with the findings of this study. When a prospective teacher gave a response, the other prospective teachers rarely interjected with another statement or question that provided mathematical support or refutation. They directed their questions and responses toward the professor. As the prospective teachers answered the professor's questions, their bodies faced her, and they only made eye contact with the professor–when eye contact was made (Field notes 1, 3/26; Field notes 2, 4/12; Field notes 3, 4/26).

Even in a class consisting of adult learners, the prospective teachers' explicit ownership in their mathematical responses was not demonstrated. The prospective teachers looked to the professor as the owner of knowledge and the authority (Sfard, 2001). They looked to her for affirmation and validation of their ideas. This perspective is consistent with uni-directional and contributory communication (Brendefur & Frykholm, 2000; Sfard, 2001). Researchers have found this perspective to also be consistent among prospective teachers faced with content-specific situations (Lo & Wheatley, 1994; Mewborn, 1999).

The direction of the prospective teachers' responses toward the professor, in addition to their body language, signify that they viewed the professor as the authority for learning in the class (Lampert, 1990; Mewborn, 1999; Williams & Baxter, 1996). The prospective teachers had looked to the professor as the person who validated their mathematical responses and not each other. Mewborn (1999) has affirmed the change that peer communication can have on prospective teachers' view on authority.

Prospective Teachers' Views on Discourse

The findings of this study reveal that prospective teachers' participation in the mathematical discourse does not align with their statements about the importance of discourse. Ann believed that mathematical talk was very important stating, "Talking about mathematics helps to reinforce knowledge; therefore, I find it extremely important. The more students, including myself, talk about any subject, including mathematics, the more learning and reinforcement will take place" (Interview 1, 9/15/2010). Beth also valued mathematical discourse in the classroom. She commented, "Talking about mathematics is very important. I have realized that even though I have always enjoyed the complexities of math, I have missed some conceptual ideas throughout my education. Therefore, the discussions that we should be having and do have sometimes allow me to connect those bridges, which we then learn how to better teach our students so that they aren't having the gaps in conceptual knowledge" (Interview 1, 9/26/2010).

While their statements place value on the importance of mathematical discourse in constructing knowledge, they did not participate in their own discourse for mathematical learning. It is possible that the prospective teachers did not realize that they had not participated in the mathematical discourse. Beth had commented, "I don't really know why we did not participate in discussions about the mathematical tasks."

Factors That Influence Discourse Participation

Avoidance strategy. The prospective teachers' level of engagement influenced their participation in each of the discourses. Engagement is a contributing factor that will be discussed in detail later in the chapter. However, avoidance strategy is a factor that may have contributed to some of the prospective teachers' lack of mathematical discourse. Jansen (2008) has described avoidance strategy as a subset of performance goals in which an individual employs strategies to hide potential discrepancies in competence with one's peers. The mathematical competency of all seven prospective teachers was not determined. The mathematical discourse required prospective teachers to make a conjecture, to take a risk. Lampert (1990) described mathematical risk taking as "requiring the admission that one's assumptions are open to revision, that one's insights may have been limited, that one's conclusions may have been inappropriate" (p. 31). Engaging in mathematical discourse requires the prospective teachers to be open for criticism, albeit constructive. It increases their personal vulnerability.

At least four of the seven prospective teachers had demonstrated mathematical ability through standardized scores; however, it could not determined that all of the prospective teachers possessed the same scores (Document, 3, 10/1). Therefore, in order to maintain safety and to maintain an image of intellectual capability, some of the prospective teachers might have chosen not to engage in mathematical discourse (Elliot & Dweck, 1988). Their choice may not have been from lack of engagement, but rather from a fear that their mathematical learning curve would be exposed (Goulding, Rowland, & Barber, 2002). Meaningful mathematical discourse requires courage and modesty. By exposing their thinking to others and exchanging ideas through talk, they might have obtained a better idea in the end.

Elimination of fear. Two possible explanations exist for the move from uni-directional communication to contributory communication during pedagogical discourse. The increase in the verbal interactions of the prospective teachers could have stemmed from the elimination of any possible avoidance strategies and their increased engagement from the discourse meeting their expectations for discussion. Unlike the mathematical tasks in which the questions asked had a single, correct response, the questions within the pedagogical discussions were open-ended and opinion-seeking. Therefore, a *right* or *wrong* response does not exist. The elimination of a correct response dissipates the notion of intellectual comparisons between peers. When there is no longer a right or wrong response, individuals do not have to fear appearing inadequate or less intelligent that anyone else. Therefore, the risk to engage in discourse decreases (Elliot & Dweck, 1988).

Goal alignment. Additionally, the increase in activity during pedagogical discourse could have been attributed to the focus on operational knowledge. Mewborn (1999) described the concern of prospective teachers about classroom life. The situations that the prospective teachers experience in their field experience are problematic for them. Mewborn characterized their concerns into four categories: classroom context and management, pedagogy of mathematics teaching, children's mathematical thinking, and mathematics content and curriculum.

In this study, the prospective teachers' experiences in managing the classroom and teaching effectively became elements for discussion as a result of being situated in a field experience at a local schools. Korthagen and Kessels (1999) discovered similar findings in their study of prospective mathematics teachers. Their increase in pedagogical discourse could have been the result of the connection between discourse and their expectation of the course. Individuals want to talk about topics they find interesting. This finding suggests that teacher educators should explicitly address the contextual situations that prospective teachers encounter during field experience because their encounters fuel their desire to engage in discourse.

The Importance of Questioning in Developing Productive Discourse

The findings of this study on classroom discourse affirm that a major component of discourse is questioning. Questioning sets the stage for discussion, provides a context for the discourse, and sustains it when the discourse is not supported by voluntary verbal interactions. These findings suggest that questioning is an important element in facilitating productive discourse.

The professor had included questioning as an instructional strategy. During their discussion on instructional strategies, the professor discussed questioning as a strategy. She presented the concept of questioning as a question-and-answer format.

> Professor: Question and Answer. You ask a question, the student will answer, there's a reaction to it. You'll ask another question either of the same student or another student. Students can go to the board; you might send a few students to the board. They could be taking notes as well. Have you seen it in your classrooms? Do you know it when you see it? How many questions should you be asking during the question and answer section of your lesson do you think?
>
> Prospective teacher: As many as it takes.
>
> Professor: That's a very good answer–as many it takes. We were talking about the listening, the different types of listening. We have to feel more comfortable maybe veering from our lesson or not being afraid of what answers you might get when you ask your students. A lot of teachers, they don't want any confusion, "Look, I got so much I have to cover, I can't go off on a tangent with my students." But you want to make sure that you are giving your students an opportunity to communicate the mathematics to them because I think that's the

> best way that you can help your students to understand, or help you understand what they understand, have them talk to you. So questioning and answering is a legitimate mode of instruction. You ask a question, one student answers, the teacher reacts, another question, which is responded to by a second student, and you just go on and on with that, like with your questioning (Audio transcription 2, 4/12).

In this conversation, the professor had described the correlation between questioning and mathematical communication. She also had expressed the importance of not avoiding the questions or answers that they might encounter during questioning. Yet in her description, the professor promotes vertical discourse in the classroom between the teacher and student rather than horizontal discourse. She relates questioning with the role of the teacher rather than promoting student questioning of peers. Through her description, she equates questioning with assessment (Wood et al., 1993). Her description of questioning describes the manner questioning was used during the mathematical discourse. During pedagogical discourse, questioning is seen as a prompt to begin conversations on general pedagogy issues.

Mathematical discourse

During classroom discourse, the professor had asked a mixture of low and high-level cognitive questions. The professor asked 78 questions during the mathematical discourse. Of those 78 questions, 50 questions addressed lower-level knowledge such as the recall of information or procedures (Audio recording transcription 1, 3/26; Audio recording transcription 2, 4/12; Field notes 1, 3/26; Field notes 2, 4/12). Krathwohl (2002) defined lower-level cognitive questions as questions that do not require an individual to *do* anything with their knowledge other than regurgitating it. He categorized these questions as *remember* and *understand* cognitive processes. Such questions include "What is degree of a right angle?" or "How do you construct a bar graph?" (Audio recording transcription 1, 3/26).

Low-level cognitive questions do not engage individuals in expressing thoughts or making justifications for their thinking (Anderson et al., 2001; Bloom et al., 1956; Krathwohl, 2002). Therefore, they are not provided a springboard that prompts them to share their thinking and deepen their current level of knowledge (Blanton et al., 2001; Cobb et al., 1997; Lo & Wheatley, 1994; Simon, 1995). This finding of the prevalence of low cognitive questioning during mathematical discourse suggests that higher-level questioning will be more beneficial to the development of mathematical knowledge construction. Questions that ask individuals to analyze, evaluate, and create require individuals *do* more with their knowledge (Anderson et al., 2001). Therefore, not only should teacher educators teach prospective teachers *about* higher-level questioning, they should immerse them in responding to higher-level questions. Their responses to higher-level questions deepens their own knowledge and aids them in fostering student knowledge.

At times, mathematical communication in the class represented a question-and-answer session especially when assessing knowledge. The following question-and-answer conversation represented interactions that occurred consistently during the mathematical discourse. The professor asked the prospective teachers questions that required a single response. However, the professor did not ask them a follow-up question to explain their response (Field notes 1, 3/26; Field notes 2, 4/12).

Professor: Oh, I wanted to correct, you know we talked a little bit about the cross-factoring and the problem that we had is, I want to kind of go over this. I'm going to actually write the problem on the board. I guess it was a problem like this: $3x^2 - 10x - 8$. And what we didn't do is we needed to multiply these two together to get the -24. That make sense [sic]? And so you were asking yourself, "What are two numbers I can, like, multiply together to get -24, but when I add those two I'm going to get -10?"

Prospective teacher 1: 6 and 4.

Prospective teacher 2: -12 and 2

Professor: Ok, -12 and +2. Remember we were saying that we wanted to try to show our students that when we get those two factors, we're really looking at area: length times width. So, let's see. We have $3x^2$. We have -8 there. And then we're going to have, I guess we could put 12x and 2x. Ok, so everything's represented, correct? Ok, so we can maybe factor out a 3x there, can we not? And what would be there to make this area $3x^2$?

Prospective teacher 1: x.

Professor: And what about here?

Prospective teacher 1: -4.

Professor: And what's going to go here if this side is x?

Prospective teacher 2: 2.

Professor: Is that right? And so our factors then become 3x, help me.

Prospective teacher 1: + 2 and x – 4.

Professor: And that's your length times your width. Ok? So just remember when your a is not 1, that you're going to have to multiply those terms together. Does that make sense to everyone? Ok (Audio recording transcription 1, 3/26).

The absence of mathematical explanations does not allow for the determination of mathematical understanding. The majority of the questions asked during the mathematical discourse did not assess conceptual learning of the mathematical concepts because the prospective teachers did not provide justifications to their responses. Therefore, only procedural knowledge can be determined based on a single response. Dewey (1933) has argued that the most important factor in preventing recitation is to make every person accountable and require exploration and justification of responses.

The lack of mathematical explanations to mathematical questions can limit even questions that are at a higher cognitive level. For example, when the prospective teachers created box and whisker plots, the professor asked them to make interpretations based on the graph.

Professor: Ok. What do you think about the data in terms of, if you're looking at your box and whiskers plot, both of them together, what do you notice about them?

Prospective teacher 1: The second has more variability.

Professor: Okay and why is that? [Pause].

Professor: Because the range is greater, same with the interquartile range. And so if you look at those two and well, and you compare them, is there one that is higher up on the, if you graphed them using the same line, is one up higher than the other one, meaning over to the right more, or how do you compare them in that manner?

Prospective teacher 1: The one without pictures is more skewed.

Professor: The measure of central tendency for the second is larger than the central tendency for the first. So it should be if you have them graphed on the same one, it should be up a little bit higher up on your, higher meaning over to your right. That's just telling you that your data for that particular group is better in terms of higher in terms of your recall was better in that situation. Make sense? (Audio recording transcription 1, 3/26).

In the above conversation, the prospective teacher did not demonstrate that she understood why her answers were correct. In fact, her pause after her response of "the second has more variability" was a potential signal that she did not conceptually understand why her answer was correct. However, she did not have to prove her conceptual understanding because the professor provided the explanation needed to justify her response. The extension of a *what* question with a *why* or *how* question transfers the ownership of learning from the teacher to the learner (Wood et al., 1993; Nathan & Knuth, 2003; Nicol, 1999).

The absence of any questions that asked the prospective teachers to justify their response may have attributed to the lack of productive discourse. In the above conversations, the professor provided them with the explanations for their responses. Williams & Baxter (1996) have advised against providing too much analytical scaffolding. Too much scaffolding prevents learners from constructing their own learning. Moreover, low-level questioning provides a potential false sense of understanding (Anderson et al., 2001; Krathwohl, 2002; Nicol, 1999). Providing a correct answer does not assure that conceptual learning has taken place. A correct answer to a problem could be achieved through an understanding of only procedures.

The discourse that had occurred during the mathematical tasks did not resemble the type of discourse the questions within the tasks promoted. Each of the tasks contained questions that prompted explanations and justifications (Document 3, 3/26; Document 4, 3/26; Document 5, 4/12). Such questions included “Explain your reasoning” or “Justify your response” (Document 3, 3/26; Document 5, 4/12). Anderson et al. (2001) characterized these types of questions as synthesis questions. Synthesis questions shift learning from recollection to a deeper level of application. Providing justifications is a consistent and pervasive element within reform-based classrooms. Therefore, prospective teachers should possess the mathematical knowledge needed to provide justifications in order to determine the accuracy of their students’ justifications. This knowledge cannot be assessed without questions that ask for prospective teachers’ explanations of their mathematical thinking.

Missed opportunities for mathematical discourse on concepts and multiple ways of thinking resulted from the prospective teachers’ lack of accountability. This findings affirms the importance of individual accountability. Hiebert et al. (1996) has suggested that personal accountability for thinking provides students with ownership of learning. Prospective teachers need to identify the various ways in which students can demonstrate personal accountability for their responses. They should also recognize the role questioning can play in providing individual accountability (van Zee & Minstrell, 1997).

Pedagogical Discourse

As compared to the mathematical discourse, questioning continued to play a prominent role in pedagogical discourse. However, the frequency of questions along with the person asking the questions shifted. Additionally, the type of questions asked during pedagogical discourse contained different characteristics than the questions asked during the mathematical discourse. This findings show that questioning can serve different purposes in discourse. It can be used not only for assessment of learning, but also for an invitation to thinking.

The findings of the study affirm that regardless of the type of discourse, questioning remains a prominent presence. During pedagogical discourse, questioning consisted of primarily open-ended questions that do not contain a single, correct response but more of an invitation to the prospective teachers to voice their own perspectives based on their experiences in the field. Their increased feelings of openness within the pedagogical resulted in their increased frequency in verbal interactions (Field notes 1, 3/26; Field notes 2, 4/12; Field notes 3, 4/26). In the following conversation, the professor had invited them to voice their perspective on homework.

Professor: How do you feel about homework?

Prospective teacher 1: I think it's the best way to make sure that you actually know the material in a math class.

Professor: What do you think about assigning it and grading it?

Prospective teacher 1: I think that you should grade it on completion. I don't think it should be a grade on accuracy.

Professor: And how do you do that?

Prospective teacher 1: Check to see if they completed it.

Professor: Ok, in what way? Do you take it up or do you walk around the classroom and you give a check or you take it up and make sure they've done 1 through 5 or they've done 5 out of 10 or…? (Audio recording transcription 2, 4/12).

The professor's question through her choice of phrasing had asked for the prospective teacher's feelings and thinking. The sharing of thinking requires a higher level of cognition because the explanation of thought involves the process of ideas. Reeves (2007) has speculated that questions that begin with how or why indicate an application of knowledge. Therefore, by asking for their opinion, the professor was promoting higher-level thinking. Moreover, by sharing their thinking, the prospective teachers engaged in higher-level thinking.

Another example of the use of open-ended questions as an invitation for the sharing of thinking occurred during the following conversation between the participants. The professor presented the prospective teachers with an open-ended question. Her question, "So anything interesting happening in your classrooms?" invited the prospective teachers to choose the topic they wanted to discuss. A prospective teacher chose to talk about the push to review for a standardized test.

Professor: So anything interesting happening in your classrooms?

Prospective teacher 1: CRCT review.

Professor: Yeah, we're going to talk about that for a minute or two. What are you observing there?

Prospective teacher 1: A big crunch to just review everything quickly and it's not teaching anymore. It's just; you need to see this one more time before you have to take a test on it.

Professor: How are you reviewing? Just warm-ups or is it consuming the whole class, consuming the entire class? How? How is it consuming?

Prospective teacher 2: Here's a worksheet, do it, let's talk about it. Here's another worksheet, do it, let's talk about it. Here's a worksheet to take home, bring it back to class. (Audio recording transcription 1, 3/26).

The environment where pedagogical discourse occurred reflected the importance of establishing comfort and safety. Comfort and safety are essential qualities to establish because they encourage participants to engage in discourse. The prospective teachers felt more comfortable asking the professor and each other pedagogical questions. This comfort had manifested when Jill asked the professor a question about writing an essential question.

Jill: I have a question about the essential question. In Wednesday's class, its Education 6200, we talked about the essential question and her big thing about it was that the essential question has to be an open-ended question. It's not allowed to have a yes or no, that's correct, that's incorrect kind of answer. How do you feel about that statement?

Professor: I would say that…

Jill: When it comes to math especially…

Prospective teacher 2: We're talking too, in school though, where like if I have an essential question like the one right here we would have points marked off on it.

Professor: When working with sets, when do I use? That's what you mean?

Prospective teacher 3: Like she would have said that this is absolutely not an essential question because there's a very specific answer to it. This is a non-essential question actually.

Professor: When working with sets, when do I use a union and where do I use an intersection? Is this an enduring understanding? Meaning, do you want your students three years from now to know the answer to that question?

Jill: Yeah, but there was a distinct, like she distinguished between an enduring understanding and essential question.

Professor: Well, an enduring understanding, I think, is going to be coming from your essential question. An enduring understanding to me, mathematically, is five years from now, I can say that, or if I were in eighth grade, when I got to high school and I had learned about quadratic function and finding their solutions then I ought to know, something that should stick with me is that when I look at a parabola it's either going to cross the x-axis or it's going to cross it one time, two times, or no times. That is an enduring understanding for me because it's something that I'm not going to forget, and it's essential that I know that in order to solve quadratic functions. Does that make sense?

Prospective teacher 2: So we're not distinguishing between enduring understanding and essential question?

Professor: Not in here.

Prospective teacher 2: That's great because we can't figure out what an essential question is (Audio recording transcription 2, 4/12).

Lo and Wheatley (1994) have attributed the ease of a response to a lack of an evaluative element associated to the question or its response. The prospective teachers did not have to fear looking less intelligent in front of each other. A right or wrong answer to any pedagogical question did not exist because the question solicited opinions. The prospective teachers did not have to utilize avoidance strategies to *save face* in front of each other.

The findings in this study regarding pedagogical discourse reveal that the prospective teachers who were interviewed do not view themselves as experts on pedagogy. Ann acknowledged the collective need for pedagogy stating, “Each of us in this course has a common concern about our own classroom management styles, and wanted to bounce ideas off of each other” (Interview 1, 9/15). The interview participants expressed that they all seemed to be at the same level when it came to pedagogy. This feeling creates a climate of acceptance and comfort to ask questions. The interview participants’ acknowledgement of their lack of knowledge about pedagogy and their engagement toward its discourse signifies that they view pedagogy as being problematic.

The Factors that Affected Engagement of Prospective Teachers in Classroom Discourse

The findings of this study indicate that the prospective teachers’ engagement levels in the discourse differed immensely. This difference depended on the nature of the discourse. The prospective teachers were less engaged in the mathematical discourse compared to the pedagogical discourse. The low engagement level of the prospective teachers during the mathematical discourse contributes to the discourse being characterized as *uni-directional*. The pedagogical discussions fostered discourse characterized as *contributory*. The prospective teachers' mathematical self-efficacy and their goals toward the course influence the difference in their engagement levels.

Mathematical Self-Efficacy

Bandura (1993) has defined self-efficacy as one's belief about whether he or she can control or regulate his or her own learning and master academic tasks. Self-efficacy is the confidence one feels about his or her own ability. Bandura stated that efficacy beliefs could influence how people feel, think, and behave. Philippou and Christou (1998) have asserted that a prospective teacher's conceptions and self-perceived relationship to mathematics are of importance in the formation of her or his learning and teaching development.

The findings of this study indicate that the prospective teachers possessed a high sense of mathematical self-efficacy. They are confident in their mathematical ability. Four of the seven prospective teachers had a quantitative GRE score that indicated strong mathematical ability and skills. Three of the four prospective teachers scored a quantitative score of 700 or higher (Document 5, 10/1). One prospective teacher scored a 560 that signified solid mathematical ability. The remaining three prospective teachers' scores were unknown because they did not grant permission to have their transcripts reviewed. Furthermore, their high mathematical self-efficacy suggests that self-efficacy beliefs were a barrier toward constructing mathematical content knowledge from mathematical discourse. All of the prospective teachers had stated that they had shared mathematical thoughts with the class rather than learning a new mathematical idea or perspective from someone else (Writing prompt 1, 3/26; Writing prompt 2, 4/12; Writing prompt 3, 4/26).

View of Mathematics. The findings indicate that the prospective teachers possess a view of mathematics that is unproblematic. Their view results from their high mathematical self-efficacy. They collectively thought of mathematics as being uncomplicated (Audio recording transcription 2, 4/12; Field notes 2, 4/12). The interview responses from the prospective teachers echoed their sense of strong mathematical confidence. Ann stated, “I think I have a pretty good understanding of mathematics. I was always good in math” (Interview 1, 9/15). Amy echoed this sentiment stating, “I feel I am strong conceptually in mathematics. I know more than just how to solve it” (Interview 1, 9/16). These findings point toward individual who choose not engage in the mathematical discourse because they feel they already know the mathematics; therefore, they do not have the need to talk about it. Their mathematical self-efficacy prevents them from gaining mathematical knowledge from the tasks they completed and their discourse.

The prospective teachers’ strong sense of mathematical self-efficacy presents contradictions with their feelings on pedagogy. Philippou and Christou (1998) have noted that efficacy beliefs could enhance or undermine performance of activities. The interview participants admitted to feelings of doubt toward their pedagogical knowledge. Ann commented that she needed reassuring in teaching areas such as classroom management (Interview 1, 9/15). Beth remarked that she felt that she needed more tangible ideas about teaching rather than mathematics (Interview 1, 9/26).

The findings of this study regarding mathematical confidence suggest that the mathematical tasks that prospective teachers participate in are not problematic for them. Lo and Wheatley (1994) suggested that making mathematics problematic is a criterion for meaningful mathematical discourse. Hiebert et al. (1996) suggested that problematic mathematics problems begin with dilemmas and questions that require inquiry and exploration. Harkness et al. (2007) likened the perspective of problematic mathematics to the concept of struggle. They described mathematical struggling as wrestling with a task where one leaves mentally fatigued, but has reached a deeper level of understanding as a result. The prospective teachers did not struggle with the mathematics within the tasks as they were presented. This suggests that mathematical situations should be presented to prospective teachers that are problematic in the mathematical content they encounter or in the way that prospective teachers interact with the situation. Assessing the prior knowledge needed for a task or identifying possible misconceptions represent problematic situations to a basic task. McNair (2000) argued that mathematical discussions should reflect an intentional effort to learn about a mathematical concept or procedure that has become problematic.

In this study, the prospective teachers had completed each mathematical task in the same manner as a middle or high school student. They followed the procedural steps that students would have been instructed to follow. As the prospective teachers completed each task, the professor had asked questions about the task and about the mathematics within the task. The prospective teachers did not ask the professor mathematical questions. Their lack of questioning signifies that the task was not problematic. Their questions during the mathematical tasks asked for clarification of procedures such as “Do you want us to label the entire circle?” or “Can you tell me how to input this into the calculator?” (Audio recording transcription 1, 3/26; Audio recording transcription 2, 4/12).

In the unit circle task, the prospective teachers went through the motions of the task. They experienced the task, but did not struggle with the task. Even (1993) found similar results in her study as prospective teachers encountered mathematical problems regarding functions that were simple and easy. She discovered that the prospective teachers did not deepen their conceptual understanding of functions because the function problems did not require the prospective teachers to think or question their knowledge of functions. Hiebert et al. (1996) stated that by allowing a subject to be problematic, it "allows students to wonder why things are, to inquire, to search for solutions, and to resolve incongruities" (p. 12).

The prospective teachers' lack of questions about the mathematical tasks or their mathematical content provides evidence that they did not find the tasks problematic. The prospective teachers did not demonstrate that they had made any connections about the relationship between the unit circle and the trig graphs because none of them stated a connection during the discourse or in their writing prompt (Field notes 1, 3/26; Writing prompt 1, 3/26). One prospective teacher remarked on the writing prompt that it was cool to see the derivation. Another prospective teacher only commented that he or she liked the task (Writing prompt 1, 3/26). However, they did not comment about the task and its mathematical connections.

The data analysis task did not resemble a problematic situation for the prospective teachers. The manner in which the prospective teachers completed the task did not align with some of the standards associated with the task. One of the standards of the task stated that "Students will pose questions, collect data, represent and analyze the data, and interpret results" (Document 3, 3/26). However, the prospective teachers did not pose questions that could be answered by the data they collected. While they did collect, represent, analyze and interpret the data, their analysis and interpretation was on a procedural level.

Similar to the unit circle task, the prospective teachers completed the task as if they were a high school student. When the professor asked the prospective teachers questions that elaborated on their current understanding, they avoided responding. During the data analysis task, the professor questioned the prospective teachers about the positioning of the median on a box and whisker plot. In the following conversation, one prospective teacher signified indifference to acquiring additional information about an idea.

> Professor: What do you guys think? I've never seen it used as less than or equal to. Who's our statistician here?
>
> Prospective teacher 1: I've never seen it as equal.
>
> Professor: It's always less than. And the reason being is because when you do a box and whisker plot sometimes your median isn't going to be part of your data. Right? Or you're, yeah, because you can have two numbers in the middle and you Have to divide by two and that number might not be a part of your data.
>
> Prospective teacher 2: But if it is, shouldn't you…
>
> Professor: You don't use less than or equal to.
>
> Prospective teacher 2: So then, what is 9 right now? Is it in the lower 50% because it's not 51%?
>
> Professor: And also…well, what do you all think? 50% of the data is less than 9.
>
> Prospective teachers: I don't know.

> Professor: I've never seen it less than or equal to for the reasons that I'm telling you. However, I am always open to other interpretations, and so this is what we'll do in this case. Let's research it and we'll come back next Monday.
>
> Prospective teacher 2: I don't care. I'm going to be honest with you. I don't have time to research anything else right now.
>
> Professor: I do. I'll do it. I'll research it. But I kind of know what I said is true, but I'm going to research it. And what I'll do is I'll post it for you (Audio recording transcription 1, 3/26).

In the above conversation, the prospective teachers had encountered a concept that was problematic for them. They did not know how to interpret a particular piece of data. When the professor charged them with researching what to do with the median, one prospective teacher stated that she did not care and did not want to do the research. The professor did not encourage the exploration of the issue. The professor first provided the prospective teachers with the interpretation and then she agreed to research the concept herself when the prospective teacher stated that she did not want to. Williams and Baxter (1996) argued that individuals learn only information that is meaningful to them. This conversation illustrates that the prospective teachers did not have to explore issues that were problematic. The conversation contradicts research that indicates problematic mathematical situations elicit curiosities and sense-making skills (Harkness, et al., 2007; Hiebert et al., 1996).

The lack of problematization of the mathematical tasks reinforces self-efficacy through the ease and success in which tasks are completed (Bandura, 1993). Birch and Bloom (2003) have found that individuals who know the solution to a problem tend to overestimate how easy it is for someone else to solve that problem. They also discovered an individual's knowledge of an issue can restrict her or his ability to understand others' perspectives on the same issue. The prospective teachers developed what Nathan and Petrosino (2003) have described as an expert blind spot, in which their easy manipulation of mathematics prevents them from analyzing the task through the perspective of one who might find it difficult.

The prospective teachers did not have to expend much effort to complete the task successfully (Field notes 1, 3/26; Field notes 2, 4/12; Field notes 3, 4/26). However, by not participating in problematic mathematical tasks that required effort, the prospective teachers did not encounter new knowledge. Furthermore, they did not participate in discourse that shaped their learning (Harkness et al., 2007). Hiebert et al. (1996) argued that the key to inviting participation in a classroom activity is to allow the mathematics to be problematic. Because the mathematical tasks were not problematic, the prospective teachers failed to link the relevance of the task to the skills and concepts they already possessed. Bandura (1993) argued that there is a difference between possessing knowledge and being able to use it under situational conditions. Hiebert et al. (1996) claimed that tasks are not inherently problematic. The manner in which one treats the task determines whether it becomes problematic.

The mathematical tasks prospective teachers encounter have the potential to be problematic if they are analyzed from varying perspectives. By varying the perspective in which the task is analyzed, prospective teachers can gain insight on the task as a *student* and as a *teacher*. Each perspective offers them valuable insight toward its implementation. The student perspective allows prospective teachers to understand the mathematics of the task. The teacher perspective allows them to understand how to implement the task and what student behaviors and responses should occur during the task. In this study, the participants did not switch from a student's perspective of *completing* the task to a teacher's perspective of *how* to present the task and facilitate student discourse *about* the task.

One way to change a task's perspective is to change the line of questioning about the task. Therefore, by asking questions about the task that requires knowledge of how to teach the task, the perspective used to look at the task changes. The three mathematical tasks did not contain questions that prompted the prospective teachers to analyze the task from a teacher's perspective (Document 3, 3/26; Document 4, 3/26; Document 5, 4/12; Document 6, 4/26). Providing questions that solicit information about the implementation of the task can be beneficial to prospective teachers. The prospective teachers did not discuss the implementation the task during or after the completion of the mathematical task (Field notes 1, 3/26; Field notes 2, 4/12; Field notes 3, 4/26). The prospective teachers used the student's lens to complete the task, but never put on the teacher's glasses to learn and understand how to present the task to students effectively. Engaging in discourse in both perspectives offers a holistic understanding of the task.

The View of Pedagogy as Problematic

Pedagogical discourse contained discussions on topics that were problematic for the prospective teachers. As one prospective teacher had commented during a discussion on writing an essential question, "Sometimes I do not feel like I know what I am doing" (Audio recording transcription 2, 4/12). Ann and Beth attributed pedagogy as being problematic to their lack of teaching experience (Interview 1, 9/15; Interview 1, 9/26).

During pedagogical discussions, the prospective teachers inquired about topics through their questions and comments, and sought the ideas and opinions of both the professor and their peers. The pedagogical discussions fostered a sense of community and contributory communication as the class collectively participated in the discourse. A conversation about discipline demonstrated their collective participation.

> Prospective teacher 1 (to Prospective teacher 2): I have a question for you. I was thinking about when I watch my teacher, the whole class always gets yelled at, and I always notice the five or eight students who have not done a single thing wrong in the class, and I feel sorry for them. Have you known any way that you can set those students apart and allow them to do fun activities without or do you just kind of have to punish the whole class too?
>
> Prospective teacher 2: I don't ever, I should say, punish the whole class. Whatever I am going to do, I'm still going to do with the class. As far as, I mean, I'll pull them out and just let them know, you know, if I'm going to give those certain students attention or whatever the case is, but I never punish the whole class.
>
> Prospective teacher 1: Because what I've noticed is like this particular class never does activities anymore, they don't do performance tasks, and it's because as a group, they can't handle them.
>
> Prospective teacher 3: I would break the class up. I would have this side working on…
>
> Prospective teacher 4: So that's acceptable, to differentiate it?
>
> Prospective teacher 2: Yeah, that's exactly what it is and I don't, now that's something else good to add to my lesson plans that I am differentiating.

Prospective teacher 1: I considered that as I was watching this class, but I didn't know if that was unacceptable.

Prospective teacher 3: I think it is perfectly acceptable and necessary.

During this conversation, the prospective teachers had explored together a topic for which they did not have answer. The above conversation consisted of four of the seven prospective teachers and did not include the professor. The conversation had a context that new learning had occurred as one prospective teacher commented that she was going to add another prospective teacher's idea to her lesson plans. This comment represented a move to reflective communication because a change in thinking had occurred (Brendefur & Frykholm, 2000). The findings of this study regarding the problematization of pedagogical topics suggests that an increased frequency of verbal statements can signal that a topic is problematic. By asking pedagogical questions, the prospective teachers seek understanding while promoting discourse. The prospective teachers had asked more questions of the professor and each other compared to the mathematical tasks. An example of a prospective teacher's question that led to an engaging discussion occurred during the following conversation when a prospective teacher had asked professor to state her opinion on establishing rules in the classroom.

Prospective teacher 1: I'm just curious to know, what is your opinion, do you feel like students should take part in the rules or not? I haven't made up my mind.

Professor: Well, I think that you should definitely. I think you should establish the rules and high expectations and let them know that.

Prospective teacher 1: But are you going to let them participate in creating them?

Prospective teacher 2: Why not let them? It would give them ownership.

Prospective teacher 3: I would let them even if they are really not. Why not guide them to establishing the rules that you would want?

Prospective teacher 2: I want them to feel like they are. That they are helping to establish the culture in the classroom.

Professor: I have even gone as far as when I was teaching, I had the students act it out, like this is how I want you to behave when you come in the classroom, this is what you do. So, don't be so quick to start teaching at the very beginning. Establish your rules at the beginning.

Prospective teacher 1: What age level is the re-enacting thing good for?

Professor: I did it in middle school.

Prospective teacher 4: And how do you feel about, this may be off the subject, but how do you feel about pre-tests for students? Because I would consider giving my kids a pre-test and go over rules all in the first day (Audio recording transcription 2, 4/12).

The prospective teachers did not appear uncomfortable asking the professor questions and advice about pedagogical issues (Field notes 1, 3/26; Field notes 2, 4/12). Not being classroom teachers yet, they might not have known the answers to pedagogical questions. They are not expected to know as one of the goals of the course is to explore pedagogical issues (Document 1, 3/26). This goal acknowledges the prospective teacher's potentially limited knowledge of pedagogy. Furthermore, this goal of seeking answers to pedagogical issues aligns with the prospective teachers' goals and expectations of the course. Regarding content, prospective teachers are assumed to possess a knowledge base.

Course expectations. Jansen (2008) asserted that the value an individual ascribes to a task is an indicator of her interest in the task. The prospective teachers held expectations toward the mathematics methods course that affected their engagement in the tasks they encountered. Ames (1992) described mastery goals as goals that focus on increasing one's understanding of content. The three prospective teachers' interview responses stated they possessed mastery goals related to pedagogy (Interview 1, 9/15; 9/16; 9/26). Beth described her mastery goal for the course as "finding new methods of teaching mathematics to students" (Interview 1, 9/26). Ann commented, "I thought the purpose of the course would be to facilitate in developing not only our own teaching methods, but also to become familiar with any background knowledge necessary in becoming an effective mathematics teacher, such as GPS curriculum and NCTM standards" (Interview 1, 9/15). Beth described her goal as receiving activities she could implement in her classroom.

> I felt that I needed more tangible ideas on teaching mathematics. I was hoping to get that in this class, because in other classes, we have talked a lot about theories of education, and I feel like I need to see how to implement those theories in the mathematics classroom. We got some of this out of the class…We were able to do some tangible activities that we can take back to the classroom" (Interview 1, 9/26).

Beth had a goal of leaving the course equipped with tangible teaching strategies. She did not state that she had a goal to deepen her understanding of mathematics. Amy also did not state that she possessed a goal to expand her mathematical knowledge. Amy commented, "I did not come into this class thinking that I needed any information to help me develop. I just figured it would help me come up with new ways of teaching different concepts" (Interview 1, 9/16). Like Beth, Amy's goal was to learn new strategies and teaching activities. She wanted to expand her pedagogical knowledge. Ann also expressed her goal for the course as a pedagogical goal.

> I was hoping to be exposed to the GPS curriculum, and the NCTM standards, both of which we were exposed to and worked with directly. It aided me in creating activities that were aligned with what students are expected to learn. I was also hoping to be able to develop lesson plans; even though I was not quite fond of the format of the assigned lesson plans, the process of designing and writing each lesson plan was very helpful in my development as a mathematics teacher (Interview 1, 9/15).

Ann's goal directly correlated to her desire to develop lesson plans and create appropriate activities. She attributed these characteristics to being a mathematics teacher. Each interview participant described having a pedagogical goal for the course and had the expectation that the course should focus on pedagogy, in particular teaching strategies. Their goals and expectations correlated to their engagement in pedagogical discourse. Because they possessed a goal to gain pedagogical knowledge, the prospective teachers who were interviewed were active participants in the pedagogical discourse.

Eccles and Wigfield (2002) affirmed the relationship between goals and interest. They have asserted that individuals' goals influence expectancies and values, which affect their interest toward a task. Ames (1992) stated that an individual's goal directly influences her perception of a task and her selection of task choice. The interview participants engaged in pedagogical discourse because it aligned more closely with their mastery goals of the course. The prospective teachers were more open to participating in pedagogical discourse. They were attentive to each other's responses–making eye contact when speaking and listening. Collectively, they participated in the discussions with more frequency and animation (Field notes 1, 3/26; Field notes 2, 4/12). The prospective teachers increased their verbal statements and questions. They exhibited facial expressions, nodded their heads while others spoke, and used their hands as they spoke.

The mathematical tasks did not align with the goals stated by the interview participants. Ann commented that she wished more time had been spent on teaching methods than doing mathematics (Interview 1, 9/15). Amy and Beth also commented on their wish that the mathematical tasks had not been a focus of the class. Amy stated, "As I have previously stated, we all have a pretty good conceptual understanding of the math; we were hoping for some instruction on teaching methods instead of more instruction on the math we already knew" (Interview 1, 9/16). Beth echoed both Ann and Amy in her statement commenting,

> I think we would have all rather focused on the teaching part of teaching math. Some of us raced through the questions in the activity. Some of us probably didn't care for whatever reason. I worry about being a good teacher. Therefore, that is what I want to know about in a methods class. I kind of feel math should be in the math class and strategies in the methods class. When you try to mix the two, one always plays a bigger part. I think the math played the bigger part and the teaching part got the short end. But, the teaching part is the part that I need the most (Interview 1, 9/26).

The relationship between goals and interest found in this study is consistent with Murphy's (2006) findings in her study of elementary school teachers. The teachers in Murphy's study did not find the relevance in studying mathematics and therefore, were not interested in those tasks as compared to others. Borko et al. (1992) found that prospective teachers wanted ideas or activities that they could easily import into a classroom. Mewborn (1999) discovered that prospective teachers reluctantly participated in mathematical problems because they did not view mathematics content as significant. The disconnect between the prospective teachers' goals and experiences affected their engagement in both the mathematical and pedagogical discourse. The absence of a mathematics goal contributed to the prospective teachers' lack of engagement toward the mathematical tasks.

Schiefele (1999) described value-related valence as a component of interest. He referred to value-related valences as the personal significance or importance one places on an activity or task. Value-related valence and goal orientation are positively correlated. The prospective teachers' lack of mathematical goals resulted in a low-value valence toward the mathematical tasks. They did not value the mathematical tasks and the related discourse because the tasks did not align with their mastery goals of developing pedagogical skills. Possessing a pedagogical goal contributed to the prospective teachers' higher level of engagement in pedagogical discourse. They possessed a high-value valence toward the pedagogical discourse.

Jansen (2008) has also found that task interest is affected by personal relevance. The more personal relevance one places on a task, the greater the interest toward the task and its completion. The possibility exists that other factors could have contributed to the lower level of engagement in the mathematical discourse compared to the pedagogical discourse because only three of the seven prospective teachers were interviewed. However, research has shown that the goals described by Ann, Beth, and Amy are consistent with the perspective commonly found among prospective teachers (Mewborn, 1999; Nicol, 1999).

Forms of Mathematical Knowledge for Teaching that Discourse Facilitated and Inhibited

The second research question of this study examined the opportunities the discourse presented the prospective teachers in their development of mathematical knowledge for teaching. The mathematics methods course is not a mathematics course; therefore, the overarching course objective is not the extension of the prospective teachers' content knowledge. Being a pedagogy course situated in a mathematical context, the course's goal was to integrate mathematical and pedagogical knowledge (Document 1, 3/26). Therefore, the development of deeper content knowledge along with their pedagogical knowledge becomes a plausible outcome.

In Hill et al.'s (2008) framework for mathematical knowledge for teaching, they defined six domains under subject matter knowledge and pedagogical content knowledge. Under the subject matter knowledge, mathematical knowledge included common content knowledge, knowledge at the mathematical horizon and specialized content knowledge. Under pedagogical content knowledge, mathematical knowledge for teaching included knowledge of content and students, knowledge of content and teaching, and knowledge of the curriculum. The mathematics methods course aimed at promoting prospective teachers' development of both mathematical and pedagogical content knowledge.

Mathematical Content Knowledge

The findings of this study indicate that the prospective teachers' mathematical discourse related to the mathematical tasks represented a superficial, surface-level understanding of mathematics that only encompasses common content knowledge. The pedagogical discourse encompassed a knowledge base of general teacher pedagogy, but was not situated in a mathematical context (Field notes 1, 3/26; Field notes 2, 4/12; Field notes 3, 4/26). Perhaps more important than what kind of teacher knowledge the discourse in the methods course facilitated was the extent to which the discourse inhibited the development of knowledge *about* mathematics and pedagogical content knowledge.

Knowledge of mathematics and knowledge about mathematics. The mathematical discourse of the methods course depicted prospective teachers who demonstrated a surface-level understanding of mathematics. Ball (1990) described the superficial understanding of mathematics as knowledge *of* mathematics. Knowledge *of* mathematics corresponded to the knowledge of mathematical concepts, ideas, and procedures. Hill et al. (2008) described this type of knowledge as *common content knowledge*, knowledge that teachers possess that is common to other professions that use mathematics. From the professor's questioning and Jill, Lisa, and Amy's questioning during the microteaching lessons, the prospective teachers demonstrated that they could perform the mathematics within the mathematical tasks. An example of the prospective teachers' knowledge *of* mathematics occurred during Amy's lesson on factoring. At the end of her lesson, she asked the prospective teachers how to factor.

Amy: How do you factor? Can one of you tell me how?

Prospective teacher 1: I noticed that you look at that last number. If it is positive, you are going to have the same sign both times. If it's negative, you're going to have different signs. So you start by putting signs in.

Amy: Beautiful. Did anyone else notice anything different from that? Or how do you factor?

Prospective teacher 2: Well, you need to find two numbers when multiplied together, they add to the number that's going to be in the middle.

Amy: Right (Audio transcription 3, 4/26).

Both prospective teachers understood the procedural steps of factoring. However, what they did not demonstrate was what Ball (1990) described as knowledge *about* mathematics, in this case, factoring. In their conceptualization of mathematical knowledge for teaching, Hill et al. (2008) described knowledge *about* mathematics as specialized content knowledge. Specialized content knowledge is "the mathematical knowledge that allows teachers to engage in particular teaching tasks, including how to accurately represent mathematical ideas, provide mathematical explanations for common rules and procedures, and examine and understand unusual solution methods to problems" (p. 378). Neither prospective teacher demonstrated that she possessed specialized content knowledge beyond the systematic procedure for factoring. This is not to say that the prospective teachers did not possess deep conceptual understanding of factoring. However, Amy did not explore her peers' knowledge *about* factoring. Her questions only solicited procedural understanding. She did not ask her peers to justify their statements about how to factor. Conversely, the prospective teachers did not offer a justification to their response.

Sherin (2002) has described a similar situation in which a teacher developed a lesson on a new mathematical topic, but only asked closed-ended questions during the lesson. In doing so, Sherin stated that the teacher’s existing content knowledge constrained her interpretation of mathematics reform recommendations and limited her ability to make changes to her practice. She surmised that teachers “either do not have enough content knowledge or that what they do know is not the right content knowledge” (p. 123).

The mathematical discourse of the tasks used in the methods course promoted common content knowledge. The questions the professor asked only required the prospective teachers to demonstrate their procedural understanding of the mathematical concepts. Their responses to the professor’s questions consisted of single-answer responses that demonstrated that they could perform the mathematical procedures. Yet, the professor did not press the prospective teachers to provide conceptual explanations or justifications. In the following conversation, the prospective teacher did not offer an explanation for her response and the professor did not press her to provide one.

> Professor: Well, what I want you to do is, I don’t care which equation. Well, first of all come up with the equation for what you see graphed there. [Pause] Ok, what did you get?
>
> Prospective teacher 1: 2/3x + 2.
>
> Professor: Umhm. How did you get that?
>
> Prospective teacher 1: Using the slope and y-intercept.
>
> Professor: Okay, she used slope and y-intercept. Is that what everyone else did?
>
> Prospective teachers: Yes (Audio recording transcription 2, 4/12).

To demonstrate a deeper understanding of the concept, the professor had the opportunity to probe the prospective teacher for a deeper explanation beyond "using slope and y-intercept" such as asking her to demonstrate how she arrived at the equation 2/3x + 2 using slope and y-intercept. Her explanation would have demonstrated that she understood why her equation was correct. Possessing knowledge *of* mathematics did not mean that the prospective teacher possessed knowledge *about* mathematics (Graeber, 1999; Tirosh, 2000). Another demonstration of the prospective teachers' knowledge *of* mathematics rather than *about* mathematics occurred during the unit circle task. The professor and prospective teachers engaged in a question-and-answer conversation about right isosceles triangles.

> Professor: All right, now let's derive the 45 degree one, and this time we're going to use the isosceles right triangle. Okay, that means that this has to be 90 degrees because it's a right triangle. And it's isosceles, which means that if that's 90, then these two have to be?
>
> Prospective teachers: 45.
>
> Professor: Right. And with an isosceles triangle, I know that two of the sides are?
>
> Prospective teacher 1: The same.
>
> Professor: Yeah, which means that this side has to be the same as that one. I'm just going to say it's 1, is that okay? It doesn't matter because they're going to reduce to whatever we get here. So, if I know that that's one and that's one, what does this have to be by way of Pythagorean?
>
> Prospective teacher 1: Square root of 2.
>
> Professor: So, now you have 45. You didn't give it to them, they derived it using

something they already know. So, what's 45?

Prospective teachers: Square root of 2 over 2.

Professor: It's really 1/square root of 2, but you rationalize it down. You can talk to them about that (Audio recording transcription 1, 3/26).

The professor had multiple opportunities to evaluate the prospective teachers' knowledge *about* right isosceles triangles through her additional questioning. However, the prospective teachers had the opportunity to provide justifications without the professor's prompting. The prospective teachers could have explained why the sides were the square root of two in terms of the Pythagorean theorem. They could have derived the value of the sides in small groups and then collectively discussed their reasoning behind the value of each side. Even and Tirosh (1995) discovered in their study that the prospective teachers knew the mathematics of certain concepts, but could not explain their knowledge to others.

Graeber (1999) has distinguished between mathematical skill and understanding. She noted that skill is what is merely memorized, can be recited or performed while understanding is what can be applied in various contexts. The prospective teachers affirmed their mathematical skills during the discourse, but not their conceptual understanding. Hill et al. (2004) cited the importance of teacher development of specialized content knowledge stating, "Teachers need to know why mathematical statements are true, how to represent mathematical ideas in multiple ways, what is involved in an appropriate definition of a term or concept, and methods for appraising and evaluating mathematical methods, representations, or solutions" (p. 27). The knowledge *of* mathematics is not enough to teach student mathematics.

The questions embedded within the mathematical tasks had the potential to demonstrate the prospective teachers' knowledge about mathematical concepts. The questions within the tasks were cognitively multi-leveled. High-level questions that required conceptual understanding existed. For example, in the polynomial task, four of the nine questions were at the *analyze* level (Anderson et al., 2001). These questions included asking the prospective teachers to explain their reasoning for their answer. In addition, the task also asked for the connections between the values on the graph, and the relationships between *x*-intercepts (Document 5, 4/12).

The prospective teachers did not demonstrate their knowledge *about* mathematics through their discourse. Even (1993) acknowledged that not knowing the *why* of mathematical concepts makes it reasonable for teachers to present students with procedures that overemphasize procedural knowledge without conceptual knowledge. Even and Tirosh (1995) admitted that it is often difficult to discern between *knowing that* and *knowing why*. Ball (1990) discovered that individuals may possess developed common content knowledge but lack specialized content knowledge needed to teach. Conversely, Ball also discovered that individuals might develop specialized content knowledge that makes him or her effective teachers but that they may lack expert knowledge of mathematical concepts.

The prospective teachers may have possessed well-developed common content knowledge of all the mathematical concepts within the tasks without specialized content knowledge. Ann, Jill, and Beth all asserted that they possessed strong conceptual understanding of mathematics (Interviews, 9/15; 9/16; 9/26). Ann explained that she possessed knowledge *about* mathematics stating, "The reason I believe that I have been successful in mathematics is that I can conceptualize the ideas, rather than just solve a problem. I would rather put a problem in context, conceptualize it, visualize what the numbers are really saying, and draw a picture" (Interview 1, 9/15). However, the discourse that the prospective teachers engaged in did not display their knowledge *about* mathematics. The mathematical discourse demonstrated that the prospective teachers contained a procedural understanding *of* mathematics.

Pedagogy: An operational focus

Each class devoted half of the class time to discourse on pedagogy (Field notes 1, 3/26; Field notes 2, 4/12). The pedagogical discourse consisted of broad topics. They included modes of instruction, homework, and discipline. The prospective teachers did not situate the pedagogical discourse in a mathematical context. However, the prospective teachers did include their experiences in their field experience mathematics classrooms as the backdrop of their questions and comments. Their field experience garnered interest in school-related issues-copy paper, student behavior, test review, and time management. The prospective teachers' interests aligned with the categories of interest Mewborn (1999) discovered in her study of prospective teachers. She described classroom management as the most important category of learning for prospective mathematics teachers. She discovered that mathematics content was the least important category of learning for prospective mathematics teachers.

The prospective teachers' concern about the realities and challenges of teaching became apparent from their conversations (Field notes 1, 3/26; Field notes 2, 4/12; Field notes 3, 4/26). The prospective teachers preferred discussing strategies for time management, learning how to keep students quiet, finding time to make lesson plans and copies, and learning how to manage the paperwork and meetings that they saw in their collaborating teachers' environments. The interview participants all acknowledged their preference for these topics of discussion (Interview 1, 9/15; 9/16; 9/25). Korthagen and Kessels (1999) cited prospective teachers' need for concrete answers to situations they encounter in schools. They described this knowledge as "action-guiding" knowledge that contrasted with the general knowledge presented in their courses (p. 5). By discussing real issues encountered in school settings, the prospective teachers developed their own knowledge based on practical situations.

Inhibited knowledge: Pedagogical content knowledge.

The call for the development of prospective teachers' pedagogical content knowledge is rich (Ball, 1990; 1991; Borko et al., 1992; Fennema et al., 1996; Hill et al., 2004; Hill et al., 2005; Hill et al., 2008; Shulman, 1986). Nathan and Petrosino (2003) described "readily-accessible" pedagogical content knowledge as a principal component of effective teaching (p. 908). Shulman (1986) defined pedagogical content knowledge as "an understanding of what makes the learning of specific topics easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons" (p. 9). Nathan and Petrosino (2003) affirmed the importance of pedagogical content knowledge for teachers by acknowledging that the possession of knowledge of a subject but lacking knowledge of how students actually learn the subject yields a perspective more aligned with the discipline than with learning processes of students. Hill et al. (2008) defined three domains that comprise pedagogical knowledge. These domains are focused on teachers' understanding of how students learn particular content.

One of the stated goals in the course's syllabus pursued the development of pedagogical content knowledge (Document 1, 3/26). The observed discourse within the mathematics methods course did not promote the development of pedagogical content knowledge. Pedagogical content knowledge was not promoted because it was neither encountered nor discussed. During the mathematical discourse of each mathematical task, the prospective teachers and the professor did not discuss the elements that encompass pedagogical content knowledge: the assumptions, prior knowledge, and misconceptions that middle or high school students may have when they embark on the task (Audio recording transcription 1, 3/26; Audio recording transcription 2, 4/12; Audio recording transcription 3, 4/26; Field notes 1, 3/26; Field notes 2, 4/12; Field notes 3, 4/26). Tirosh (2000) asserted that " knowing without understanding why in terms of teacher's knowledge of students' ways of thinking is no more meaningful that knowledge in the context of mathematics content" (p. 22). Even (1993) claimed that pedagogical content knowledge determines the questions teachers ask, activities they design, and suggestions for learning they give to students. Without a rich base, prospective teachers have limited ability to do the above effectively.

Knowledge of content and students. Hill et al. (2008) listed *knowledge of content and students* as a domain within pedagogical content knowledge. They defined *knowledge of content and students* as the knowledge a teacher possesses about the ways in which a student encounters mathematical concepts and tasks. They likened this to a teacher's familiarity of student errors or misconceptions. They insisted that teachers must be able to examine and interpret the mathematics behind student errors. Even and Tirosh (1995) also included the sources of conceptions, misconceptions, and ways of thinking as components of knowledge of students. Hill et al. (2004) included the knowledge about what makes a concept easy or difficult for a student to comprehend as knowledge of students.

In each of the mathematical tasks the prospective teachers encountered, the development of *knowledge of content and students* had the opportunity to occur. In the data analysis task, the prospective teachers had the opportunity to explore the misconceptions middle school students may have toward mean, median, and mode through the professor's questioning (Field notes 1, 3/26). The professor asked the prospective teachers, "What type of knowledge do students need to have before they can create a box and whisker plot?" (Audio recording transcription 1, 3/26). Her question asked the prospective teachers to determine the prior knowledge middle school students need to complete the task successfully. The prospective teachers responded with mean, median, mode, and drawing graphs. However, the prospective teachers did not explain *why* middle school students would need to know these concepts to understand box and whisker plots.

In the polynomial tasks, the prospective teachers had the opportunity to define and explain common errors high school students make when determining equations and their graphs (Field notes 2, 4/12). This opportunity presented itself when one prospective teacher admitted that she had graphed the equation incorrectly. The prospective teacher had the opportunity to explain what she did that was wrong. The prospective teachers could have engaged in a discussion about what misconception led to the error and what other errors should be anticipated. Graeber (1999) acknowledged that tapping into existing ideas is the first step in helping to correct conceptual errors. Shulman (1986) noted that “if students’ preconceptions are misconceptions, which they so frequently are, teachers need knowledge of the strategies most likely to be fruitful in reorganizing the understanding of learners” (p. 9).

Emphasizing common misconceptions has been proven powerful for the development of mathematical pedagogy (Even & Tirsoh, 1995; Tirsoh, 2000). The participants missed opportunities to engage in discourse that developed the prospective teachers’ knowledge of mathematics and students. A mathematics methods course provides an optimal environment to explore the prior knowledge, assumptions, and misconceptions students bring to a mathematical task. Through the exploration of students as learners, the prospective teachers develop a knowledge base on how to assess prior knowledge and rectify misconceptions. Murphy (2006) included the knowledge of learners as part of the “social activity of teaching” (p. 229). As Simon (1995) has noted, what the prospective teachers make of the task and their experience with it determines their potential for learning.

Knowledge of content and teaching. Hill et al. (2008) listed *knowledge of content and teaching* as the second domain under pedagogical content knowledge. They defined *knowledge of content and teaching* as the knowledge that a teacher possesses about *how* to teach a particular concept. Shulman (1986) described this knowledge as the representations that are most useful for teaching specific content. This type of knowledge represented a teacher knowing how to present a concept to students using multiple representations or being aware of alternate methods that can be used to arrive at a solution. Graeber (1999) expressed the need for alternative representations stating, “There are different logical or experiential paths that lead to the same ideas; similar experiences may lead to different yet valid ideas; different models help different students construct ideas, different students make different connections and different numbers of connections of ideas" (p. 202).

Similar to the absence of the development of *knowledge of content and students*, the prospective teachers did not engage in discourse that promoted the development of *knowledge of content and teaching*. The goals that Jill, Lisa, and Amy all stated that they had for the course related to the development of *knowledge of content and teaching* (Interviews, 9/15; 9/16; 9/26). All three prospective teachers commented that they wanted multiple strategies for teaching mathematical concepts. Amy commented that "activities on how to teach math effectively" was essential teacher knowledge (Interview 1, 9/16). Beth echoed Amy's comment stating, "The strategies that I can implement to get students to learn math and really learn it and not forget it after the unit is over is essential" (Interview 1, 9/26).

However, the mathematical discourse did not promote the development of knowledge of content and teaching. The prospective teachers did not discuss the multiple ways that the tasks could be represented such as using manipulatives, or story problems (Ball, 1990; Even & Tirosh, 1995). The prospective teachers encountered mathematical tasks and engaged in discourse that allowed for the opportunity to discuss alternate representations. During the data analysis task, the professor asked the class, "What different ways could you represent the data from our activity?" (Audio recording transcription 1, 3/26). One prospective teacher responded to her question stating "A double bar graph." The prospective teacher did not explain or justify her choice of representation. Furthermore, none of the other prospective teachers offered another representation. Their lack of response signified that the other prospective teachers were either unengaged in the discourse and did not want to answer or that they did not know of another representation for the data.

The polynomial task had the potential to engage the prospective teachers in discourse on knowledge of content and teaching. During the task, the professor introduced the use of the graphing calculator as a tool to visualize the graphs. Some of the prospective teachers did not use the calculator, preferring to draw the graphs by hand (Field notes 2, 4/12). By recognizing that they collectively were completing the task using two representations, the prospective teachers had the opportunity to compare and contrast the success of the task using the two representations.

The lack of discourse regarding alternate representations might have been attributed to the prospective teachers' lack of knowledge regarding alternate representations. Graeber (1999) acknowledged that a lack of knowledge of alternate representations by teachers prohibits them from viewing mathematics and mathematical tasks in multiple ways. The prospective teachers might not have known other ways to represent the mathematics within the mathematical tasks. They may have known only one way to represent the mathematics. However, the prospective teachers' mathematical discourse did not examine whether there were alternate methods that could have been used to solve the task. Therefore, the depth of the prospective teachers' knowledge of multiple representations and alternate solution methods remained unknown.

As with *knowledge of content and students*, the opportunity for the development of *knowledge of content and teaching* did exist within the course. The microteaching lessons provided a context for the development of alternate representations as the prospective teachers presented lessons on various mathematical concepts (Field notes 3, 4/26). The benefit of microteaching lessons is to provide different ways of teaching mathematical concepts to students. As Jill, Lisa, and Amy presented their lessons, they presented the prospective teachers with a new way of teaching and representing the concepts of volume, probability and factoring. However, their discourse about the lessons did not promote the understanding of the relationship between content and teaching. The professor asked each of the prospective teachers to comment on the instructional lessons-something they liked and something the prospective teacher could improve on. The prospective teachers did not offer mathematical comments about the lessons (Audio recording transcription 3, 4/26; Field notes 3, 4/26). After Jill's lesson on probability, the prospective teachers commented on her lesson.

Professor: What did you really like about Jill's lesson?

Prospective teacher 1: It was hands-on.

Prospective teacher 2: I like the fill-in-the-blank notes a lot. When I do mine, I usually make them like really fill-in-the-blank, you know like a sentence or word missing. This is so much better because it just ask a question and then they fill it in.

Prospective teacher 3: I liked the manipulatives, the die and the quarters. I thought that was fun.

Professor: Anyone else?

Prospective teacher 4: I thought the whole lesson was easy to follow, with good directions, and the recording instrument was good. It flowed nicely.

In the conversation above, only one prospective teacher commented on the manner in which the mathematics was represented. She commented on Jill's use of manipulatives. The prospective teachers had the opportunity to launch into further discussion on the use of manipulatives as an alternate representation. However, the other prospective teachers did not acknowledge that they recognized Jill's use of manipulatives as an alternate representation for probability (Field notes 3, 4/26).

The mathematical discourse on *knowledge of content and students* and *knowledge of content and teaching* certainly might have existed in other class meetings that were not observed. However, the observed discourse of the prospective teachers did not demonstrate that these types of pedagogical content knowledge were topics of discussion. The absence of discourse related to these two types of pedagogical content knowledge limited the prospective teachers' development of teacher knowledge.

Concluding Remarks

The findings of this study suggest the existence of conditional elements that affect classroom discourse. These conditional elements include knowledge and interest. However, findings indicate that discourse can occur with the existence of one of these elements. The prospective teachers participated in mathematical discourse because they possessed mathematical self-efficacy even though they lacked interest in the mathematical discourse. Their participation was minimal and lacked characteristics of reflective and instructive communication. Their mathematical knowledge base allowed them to respond to mathematical questions. The relationship between mathematical self-efficacy and actual mathematical knowledge could not be determined from the discourse.

Their level of engagement in mathematical tasks prevented the discourse from becoming meaningful and a tool for learning. This suggests that discourse can occur among individuals that possess a knowledge base in the topic even if they do not possess goals or motivation toward the topic.

The prospective teachers participated in pedagogical discourse because they possessed an interest in pedagogical topics even though their self-efficacy was weaker. The incorporation of pedagogical discourse aligned with their stated mastery goals and course expectations. Research has affirmed prospective teachers' perspectives of methods courses as those that teach instructional strategies. The interest generated from their goals and expectations produced higher levels of engagement even though their knowledge on base on pedagogical issues was limited. This suggests that motivation can drive discourse participation even if knowledge possession is limited. Both discourses lacked one of the conditional elements that restricted the discourse from possessing qualities that exhibit a powerful collective construction of knowledge. This suggests that both elements *together* can provide an optimal environment for *meaningful* discourse.

The findings of this study regarding participation in discourse reveal that the existence of one or both of these elements does not guarantee participation. The existence of possible avoidance strategies represent the possibility that a prospective teacher might have possessed interest toward the mathematical tasks but lacked self-efficacy. Therefore, her lack of self-efficacy could have prevented her from participating in the discourse from fear of appearing less intelligent to her peers. Likewise, a prospective teacher could potentially possess both elements and choose not to participate because of personality traits such as shyness or fear of public speaking. Furthermore, conditional factors such as having a bad day at work, personal issues, or health could prevent an individual from participating in the discourse. These factors suggest that an individual's participation and engagement in discourse is fluid. The engagement level can change from class to class depending on the combination of conditional factors that exist during that period of time.

The findings of this study regarding mathematical communication of a selected group of prospective teachers reveal that neither type of discourse exhibits characteristics of high communication levels. The characterization of the discourse cannot not imply that learning did not exist. It can only imply limited opportunity for learning. Research has correlated knowledge construction with high communication levels. The prospective teachers might have gained mathematical and pedagogical knowledge but did not state so explicitly. The prospective teachers did not provide any verbal or written evidence that meaningful learning had occurred as a result of the classroom discourse. This indicates that teacher educators should ask explicit questions or engage prospective teachers in specific tasks that assess the amount of gained knowledge.

Finally, the findings of this study on classroom discourse indicate that an integrated approach to mathematical teaching for learning can be problematic. The course attempted to promote mathematical pedagogical knowledge through the immersion of mathematical tasks. This approach conflicted with the prospective teachers' expectations about methods courses. Therefore, teacher educators of methods course should explicitly explain the purpose and instructional approach of the methods course. As a result, prospective teachers will understand the purpose of an integrated approach. By engaging prospective teachers in conversations about mathematics using both a student and teacher perspective, they can develop knowledge *of* the mathematics and *about* the mathematics.

Limitations of the Findings

Several obstacles limited the study's findings. First, the sample size of the study was small. The participants consisted of seven prospective teachers and one professor. The number of students in the class predetermined the sample size. The semester only offered one initial mathematics methods course for MAT students. The significance of the course toward prospective teachers' preparation for classroom teaching determined its selection for the study. Consequently, this selection created an additional limitation. The methods course selected was only for prospective mathematics teachers. Therefore, the results of the study cannot be transferred to other academic disciplines.

An additional limitation to the study included the time I had available to observe and collect data. My data collection occurred midway through the semester. I did not know the content of the mathematical discourse of the previous class sessions. Past discussions could have had a potential impact on future mathematical discussions including the discussions that I observed. Furthermore, the nature and purpose of previous discussions could have been different from the observed discussions. Therefore, not observing discourse of a particular nature did not necessarily mean that it did not exist. The same could have been true about the pattern of discourse. Furthermore, the prospective teachers' engagement in the discourse could have potentially been higher or lower in previous class meetings.

Another limitation included the prospective teachers' inconsistency in completing the writing prompt. Each class, all of the prospective teacher received a copy of the specific web address for the prompt along with a thank you incentive of a candy bar to entice them to complete the prompt. However, not all of the prospective teachers completed the prompt. For the first class meeting, only five of the seven completed the prompt, the second class meeting, four of the seven completed the prompt, and for the third class meeting, four of the seven completed the prompt.

The presence of an observer could have limited the quality and authenticity of the discourse within the classroom. Merriam (2009) noted that some people become nervous or timid when it is known that discussions are being recorded. Even though there no identifiable qualities other than different voices that distinguish different participants existed, the knowledge that an outsider is observing and listening to class conversations could have presented a limitation on the nature of the discourse.

Finally, the possibility of researcher bias existed. As a former mathematics teacher, I entered the study with my own set of beliefs and understandings regarding discourse and mathematics pedagogy. Furthermore, as a school administrator, I acknowledge having an additional lens of one who understands the type of skill set that is required of prospective teachers as they enter the classroom. I am aware of the calls for change within the school setting and the topics that schools are focusing on with respect to the professional development of teachers. Therefore, I looked for the current topics in professional development, such as questioning, to emerge as a theme in the data. However, I do not think I should apologize for having two hats, one of a teacher and one of a public school administrator. I believe that my dual roles enhanced my perspective of the discourse and the topics within the discourse. Even though someone else may have interpreted the data differently, I feel that my experiences and skills have allowed me to examine the discourse in a comprehensive way.

CHAPTER VI

SUMMARY AND IMPLICATIONS

Summary of the Study

Purpose

Mathematics reform efforts have generated interest in the way mathematics is taught and the level of knowledge mathematics teachers possess (Ball, 1990; Even & Tirosh, 1995; Hill et al., 2008; Hill et al., 2005; Shulman, 1986; Simon, 1995; Tirosh, 2000). Furthermore, reform efforts have advocated the perspective that mathematics should be taught in a community environment rather than in an environment of isolated individuals (Cobb et al., 1997; Simon, 1995). Facilitating a community of learners requires prospective teachers to receive training in mathematical discourse. In order to facilitate purposeful discourse, prospective teachers need the skills to identify, engage in, and value discourse themselves. Reform researchers have also analyzed the mathematical knowledge that prospective teachers possess and have called for the increased attention to content and pedagogical knowledge (Ball, 1990; Even & Tirosh, 1995; Hill et al., 2008; Hill et al., 2004; Shulman, 1986; Simon, 1995; Tirosh, 2000). Teacher educators have been charged with teaching prospective teachers mathematics at a deep, conceptual level along with specific pedagogical skills that are mathematics-specific. The purpose of this study was to examine the manner in which prospective mathematics teachers in a graduate mathematics methods course engaged in discourse. Additionally, it investigated the forms of mathematical knowledge for teaching that the discourse facilitated.

The participants selected included prospective mathematics teachers in a graduate MAT program due to their possession of a degree in mathematics or a mathematics-related field and my interest in mathematics education stemming from my previous work as a middle school mathematics teacher and my current position as a middle school administrator.

Two primary research questions guided my study on classroom discourse. The first research question focused on the existence of classroom discourse. How did discourse appear within the instructional environment of a mathematics methods course for prospective teachers? Specifically, what were the patterns, functions, and nature of classroom discourse? Furthermore, what role did questioning play in shaping classroom discourse? This study sought to investigate the manner in which prospective teachers participated in classroom discussions as learners. The second research question that guided the study asked about the opportunities prospective teachers were provided through the discourse to develop mathematical knowledge for teaching. The study sought to determine whether discourse provided a vehicle for the construction of teacher knowledge.

Theoretical Perspectives

Mathematics education researchers have become aware of the importance of mathematical conversation for successful student learning (Blanton et al., 2001; Cobb et al., 1992; Cobb et al., 1997; Sfard, 2001; Simon, 1995; Williams & Baxter, 1996). Communication has become a focal point of mathematics reform because it is through meaningful conversations about mathematics that students construct meaning (Simon, 1995). In this study, theories of communication were used to characterize classroom discourse (Brendefur & Frykholm, 2000; Cobb et al., 1997; Hatano & Inagaki, 1991;

Williams & Baxter, 1996). These theories of communication establish vertical discourse as discourse that is dominated by the teacher and flows in a vertical direction between teacher and student as communication (Brendefur & Frykholm, 2000). As a result, fewer opportunities exist for construction of learning. Higher levels of communication and learning are the result of horizontal peer discourse among students with the role of the teacher being a facilitator of learning.

In addition, forms of teacher knowledge have become a topic of interest of mathematics education researchers as they seek to determine specific teacher knowledge needed for effective mathematics teaching (Ball, 1990; Hill et al., 2004; Hill et al., 2008; Shulman, 1986). I utilized Hill et al.'s (2008) conceptualization of knowledge for mathematics teaching as a framework to analyze the forms of teacher knowledge of the prospective teachers. Their framework establishes mathematical content knowledge and mathematical pedagogical knowledge as essential knowledge with three sub-categories under each type of knowledge.

During my data collection, I realized how questioning played a dominant role in the shaping of classroom discourse. The questions asked during classroom discourse shaped the depth of the responses given and the flow of the discourse. I also found that the type of questions varied depending on the topic of discussion. Therefore, my theoretical perspective included a framework for conceptualizing questioning. I relied upon the work of Anderson et al. (2001) to analyze the nature of the questions within the classroom discourse. They reconceptualized the cognitive taxonomy originally established by Bloom et al. (1956) by expanding their cognitive processes and dimensions. Their revised taxonomy conceptualizes questions of lower cognitive ability as those that only solicit recall and regurgitation of information (Anderson et al. 2001). Questions that are of higher cognitive ability require application and synthesis of knowledge to other contextual situations.

From the data, I also determined that another key factor in classroom discourse and the acquisition of teacher knowledge included the prospective teachers' engagement level. They exhibited varying levels of engagement depending on the topic of the discourse. I drew upon the work in the areas of self-efficacy, achievement theory and motivation to analyze the factors that affected their engagement (Ames, 1992; Bandura, 1993, 1997; Elliott & Dweck, 1988; Harkness et al., 2007; Hiebert et al., 1996; Jansen, 2008; Schiefele, 1999). Bandura (1993) linked engagement to self-efficacy beliefs and motivation. Engagement is found to be greater when individuals are presented tasks that are problematic in nature (Harkness et al., 2007; Hiebert et al., 1996). Engagement is also influenced by the expectations and goals of individuals (Ames, 1992; Elliott & Dweck, 1988; Jansen, 2008; Schiefele, 1999). Higher levels of engagement and motivation of tasks result in the close alignment of expectations and goals associated with the task.

At the outset of this study, I made three propositions concerning the discourse of the prospective mathematics teachers. The first proposition I made was that the prospective teachers would engage in discourse with each other and that their discourse would result in self-reflection. The second proposition I declared was that there would be an unequal distribution of time toward developing mathematical content and mathematical pedagogical knowledge. The final proposition I made was that the purpose of the discourse for the prospective teachers in the methods course was to gain or clarify pedagogical knowledge.

Methodology

The participants selected for the study included seven prospective teachers and their professor. Convenience sampling determined their selection. Data collection employed the use of multiple data sources. These included a classroom observation protocol, an interview protocol, relevant documents pertaining to the mathematical tasks observed in the course, academic transcripts of prospective teachers, a reflective writing prompt, and audio recordings of the classroom discourse for each class meeting. Data collection took place during three class meetings. Each class meeting lasted approximately three hours. In each class, I collected documents related to observed mathematical tasks, gathered field notes, and audio recorded the discourse of the class. After each class, I asked the prospective teachers to complete the writing prompt anonymously online through Survey Monkey.

Analysis of the data was initially completed by hand through line-item coding. Established coding schemes developed by Nathan & Knuth (2003) along with a questioning coding scheme based on Anderson et al.'s (2001) cognitive process dimensions were used to code the data. During the hand coding, I decided to incorporate the NVivo software as a means of data analysis. Therefore, I abandoned the hand coding and coded the data using NVivo software. After open coding was completed, I established similar categories from axial coding and then determined core categories from selective coding.

After this first data analysis, I developed some hypotheses that I wanted to test against further data. Therefore, I decided to create an interview protocol. I asked all seven prospective teachers to participate and three responded to my request and participated in the interview. The three prospective teachers participated in the interviews approximately four months after the completion of the course. I sent the three prospective teachers the interview protocol which consisted of six pre-determined questions via electronic mail. Based on their responses, I submitted four additional questions.

Upon the completion of the coding of the interview responses, I re-evaluated my data and developed emerging themes. I tested these themes against the data and searched for alternate explanations. Member checking with the professor, a case study protocol, and triangulation ensured credibility and dependability. I sent the professor of the course my findings for her review to ensure that the findings were accurate and reported with fidelity. An audit trail of the data collection and analysis established a case study protocol. The use of multiple data sources supported triangulation of the findings.

Limitations presented themselves during this study. One limitation included the number of participants in my sample. The small sample also proved to be a limitation in the implementation of the written prompts as few of the prospective teachers completed the prompts. An additional limitation to the study included the timing of the data collection. I began my data collection during the middle of the semester. Therefore, I did not observe previous classes and the discourse that occurred. A final limitation was that the interviews conducted with the three prospective teachers were conducted four months after the mathematics methods course had ended.

Findings

The discourse of the prospective mathematics teachers did not reflect the discourse advocated by mathematics reform efforts. The discourse did not reflect Brendefur and Frykholm's (2000) higher levels of communication. Additionally, the discourse addressed teacher knowledge as two separate entities–mathematics and pedagogy, as opposed to one integrated unit. Therefore, differences existed in the characteristics of each type of discourse. The characteristics of the mathematical discourse and the pedagogical discourse in which the prospective teachers participated is summarized in Table 14.

Table 14

Summary of Characteristics of Mathematical and Pedagogical Discourse

	Mathematical Discourse	Pedagogical Discourse
Pattern	Uni-directional communication Vertical flow between professor and prospective teachers	Contributory communication Vertical flow with some integration of horizontal flow between prospective teachers
Nature	Mathematical tasks	General teacher pedagogy and operational issues
Questioning	Assessment of mathematical knowledge Closed-response Majority low-level cognition of recall	Prompts for discussion Open-ended Opinion-based Higher-level cognition of application
Engagement	Low High self-efficacy beliefs Not aligned with expectations	High Lower self-efficacy beliefs Aligned with expectations

The mathematical discourse that occurred revolved around the mathematical tasks the prospective teachers completed. The discourse resembled traditional, vertical discourse. The prospective teachers viewed the professor as the disseminator of information. She dominated the discussions providing more verbal statements than the seven prospective teachers combined. The flow of the discourse was vertical occurring between the professor and the prospective teachers. The communication level of the mathematical discourse was characterized as *uni-directional*. The prospective teachers did not engage in discourse among themselves on mathematical topics.

The professor asked the prospective teachers many questions. During mathematical discourse, these questions were used to assess their mathematical understanding at a surface level. The majority of the questions consisted of low-level, closed-response questions. The questions assessed their knowledge *of* mathematics rather than their knowledge *about* mathematics (Ball, 1990; Hill et al., 2008). The responses provided by the prospective teachers were succinct and without elaboration.

The professor did not probe the prospective teachers for explanations of their mathematical responses, instead she provided the mathematical explanations to their responses herself. The mathematical scaffolding provided by the professor often took the place of the explanations and justifications that prospective teachers could have provided. Thus, the depth of mathematical knowledge possessed by the prospective teachers could not be determined. During microteaching lessons, three prospective teachers taught mathematics lessons, that possessed the same qualities as the mathematical discourse in the class.

The engagement level of the prospective teachers toward the mathematical tasks was low. Their feelings of mathematical self-efficacy, lack of problematic situations, and disconnection between the course and their expectations influenced their lack of engagement. The prospective teachers maintained high self-efficacy toward mathematics. Previous mathematical experience in courses supported their feelings of self-efficacy. Viewed transcripts reflected individuals who possessed high mathematical scores in course work and in standardized tests. However, I was not able to view the transcripts and GRE scores of all the prospective teachers. Therefore, I could not establish empirical support for all of the prospective teachers' mathematical self-efficacy. The prospective teachers who were interviewed expressed their mathematical confidence. Furthermore, the prospective teachers expressed their mathematical confidence during their discourse. Their high levels of mathematical self-efficacy contributed to their lack of engagement in mathematical discourse because they felt that they knew how to do mathematics. Therefore, they did not need to talk about it.

The possible existence of avoidance strategies provided one explanation for the prospective teachers' lack of engagement. All of the prospective teachers professed their self-efficacy through their words or body language. Yet, it remained possible that some of the prospective teachers did not engage in mathematical discourse because they did not want to appear less intelligent to their peers.

The lack of struggle the prospective teachers faced in completing the tasks also influenced their self-efficacy and engagement. They did not view the mathematical tasks as problematic. They completed the tasks through the perspective of a student. The mathematical discourse did not challenge them to make conjectures or question their mathematical thinking. Because they did not perceive the mathematical situations as problematic, the prospective teachers did not report having learned from the mathematical tasks. They also did not view mathematics content as a goal or expectation of the course. Interview participants’ responses stated that they hoped to learn teaching methods from the course, not mathematics content. A disconnect existed between the prospective teachers’ stated goals and the mathematical tasks because they did not view the tasks through a teacher perspective. Consequently, no opportunity presented itself for pedagogical content knowledge to develop during the mathematical tasks.

The pedagogical discourse in the class explored topics such as instructional methods, lesson planning, and classroom management. Operational issues interested the prospective teachers as discussions also included topics such as grading homework, making copies, and time management. The pedagogical discourse contained characteristics of contributory communication that signaled a move toward a community of learners. Unlike with mathematical discourse, the prospective teachers participated in horizontal discourse with their peers during pedagogical discourse. The professor maintained a primary role in the discourse but the prospective teachers were active participants in the discourse. They provided explanations of their responses, drawing from their field experience in mathematics classrooms.

Questioning became a prominent element of pedagogical discourse. The prospective teachers asked the professor and each other questions that solicited others’ opinion. Questioning became a springboard for discussions because the questions were open-ended and opinion-based. These questions easily solicited responses from prospective teachers because they did not fear providing a right or wrong response. The prospective teachers did not have to employ any avoidance strategies for fear of appearing less intelligent to others because a *right* or *wrong* answer to the pedagogical questions did not exist. They did not worry about stating a correct response because the questions solicited opinions. The prospective teachers freely voiced their perspective without the need to feel competent among their peers. Both the professor and the prospective teachers provided scaffolding on pedagogical topics. The prospective teachers valued the ideas provided by their peers.

The prospective teachers' participation in the discourse demonstrated their high level of engagement during pedagogical discussions. The prospective teachers admitted to weaker feelings of self-efficacy in the pedagogical domain than in the mathematical domain. According to the prospective teachers who were interviewed, their lack of pedagogical self-efficacy influenced their need for pedagogical knowledge. Interview participants placed a high utility value on pedagogical knowledge. They described pedagogical knowledge as required knowledge toward their development as teachers. The interview participants stated that their mastery goal was their development of pedagogical knowledge. The prospective teachers met their expectations of the course when they discussed pedagogical issues. Interview participants all described expectations of gaining knowledge about teaching. Their outcome expectancy and mastery goal linked together when the prospective teachers engaged in discourse on pedagogical knowledge.

The discourse in the mathematics methods course inhibited the development of deep mathematical content knowledge and pedagogical content knowledge. The prospective teachers demonstrated their mathematical content knowledge was at a surface level based on their discourse. They overlooked opportunities to cultivate conceptual knowledge through the mathematical tasks. Mathematical or pedagogical discourse did not address pedagogical content knowledge. The prospective teachers did not engage in discourse that discussed teaching mathematics to students using multiple representations or providing alternate solution methods. Furthermore, the prospective teachers did not explore the conceptions or misconceptions students may possess when completing mathematical tasks. The discourse did not promote the prospective teachers' development of a teacher perspective toward mathematics or the mathematical tasks they encountered.

Conclusions

Based on the findings of the study, I provided the following answers to my research questions:

1. How does discourse appear within the instructional environment of a mathematics methods course for prospective teachers?

The discourse within a mathematics methods course varied depending upon the nature of the tasks involved and the topics of discussion. Mathematical tasks solicited discourse that was primarily uni-directional with little peer discourse. Brendefur and Frykholm (2000) describe uni-directional communication as limiting to the meaningful construction of knowledge. Pedagogical discussions solicited discourse that remained vertical but contributory as the prospective teachers moved toward horizontal discourse. While the level of communication was contributory, the discourse still lacked the reflective qualities that Brendefur and Frykholm identified as necessary for meaningful learning. Analytical scaffolding and questioning were dominant features of both types of discourse. In mathematical discourse, questioning primarily served as a means of assessing mathematical understanding of concepts. The frequency of low-level questioning facilitated procedural learning of mathematics. The professor's scaffolding of mathematical responses limited the prospective teacher's accountability of their own thinking.

2. How did the discourse in the mathematics methods course facilitate or hinder opportunities for prospective teachers to develop mathematical knowledge for teaching?

Hill, et al. (2005) characterized mathematical knowledge for teaching as including "explaining terms and concepts to student, interpreting students' statements and solutions…[and] using representations accurately in the classroom…" (p. 373). The discourse in the mathematics methods course about the mathematical tasks did not focus on how students might interpret and approach the task, how to use the task with students to build conceptual understanding, how to adapt the task for particular learners' needs, or other such pedagogical content issues. Rather, the discourse during the mathematical tasks pertained to procedural aspects of doing mathematics.

Even the pedagogical discourse in the methods course revolved around broad, general pedagogy, or operational knowledge of classrooms, such as behavior management. Opportunities were not provided for the prospective teachers to reflect upon and discuss how the mathematical tasks they worked on in their methods course would play out in 6-12 classrooms. In other words, opportunities were not provided for the prospective teachers to develop the mathematical knowledge for teaching that would prepare them "to carry out the work of teaching mathematics" (Hill, et al., 2005, p. 373).

Implications For Practice

Reform initiatives in mathematics education have invited teachers to create learning environments that foster learning by allowing learners to explore mathematical ideas through communication (Brendefur & Frykholm, 2000; Borko et al., 1992; Cobb et al., 1997; Nathan & Knuth, 2003; NCTM, 1991, 2000; Simon, 1995; Wood et al., 1993). However, this study demonstrates that mathematical discourse in a mathematics methods course is limited and narrow. Implications for practice and research include steps that can be taken by teacher educators to promote mathematical discourse as a vehicle for teaching and learning.

Cooney (1994) suggested that teacher educators recognize that their students' beliefs and practices may not be aligned with those of teacher education programs. Brendefur and Frykholm (2000) advised teacher educators to understand prospective teachers' conceptions of mathematical communication and their belief structures at the beginning of the teacher preparation process. This study presents several implications for teacher educators to consider concerning practice.

This study showed that the prospective teachers were not presented with mathematics that they found to be problematic. As a result, they exhibited a lack of engagement and consequently, a lack of discourse surrounding the mathematical tasks. Additionally, the lack of problematic encounters with mathematics inhibited the prospective teachers' continuing growth of mathematical knowledge. Even (1993) has advocated the immersion of prospective teachers in an environment in which powerful constructions of mathematics can occur. Undoubtedly, prospective mathematics teachers encounter problematic situations within their mathematics courses. However, problematic mathematical situations should not be limited to mathematics-content courses. Changing the perspective in which a mathematical task is analyzed can present mathematics in a problematic manner. For perspective teachers, this could be viewing a mathematical task through a teacher's perspective.

Teacher educators can find benefits in presenting prospective teachers with mathematical situations that are problematic. Hiebert et al. (1996) discovered that when mathematics is problematic for learners, learning occurs. Tirosh (2000) asserted that learning occurs when an individual's conceptions are challenged. Both of these assertions are consistent with the findings of the present study. The prospective teachers did not learn or alter their thinking of mathematical concepts because the tasks they encountered did not produce an intellectual struggle (Harkness et al., 2007). Therefore, they did not see the mathematical purpose of the task. Lampert (1990) has described the ability to make and test mathematical hypotheses as the most important criterion in selecting a mathematical problem for an individual to encounter.

Prospective teachers need to encounter mathematical situations in which they struggle mathematically, but gain pedagogical skills from the struggle. By engaging in a mathematical struggle, prospective teachers can gain mathematical content and pedagogical content knowledge. The aim for mathematics teacher education courses should be to develop and deepen both forms of knowledge (Goulding et al., 2002). Middle and high school mathematics can be problematic for prospective mathematics teachers in the way they interact with the mathematical concepts. Through deep investigations and rich conversations, prospective teachers have the opportunity to expand their knowledge of what may be considered a basic mathematical concept.

The disconnect between the prospective teachers' expectations and experiences in the class contributed to the lack of purposeful mathematical discourse. An expectation of the prospective teachers was to gain teaching methods that would aid them in teaching mathematics. They did not see that their expectation had the potential to be met through their participation of mathematical tasks. Von Minden, Walls, and Nardi (1998) asserted that in order to understand mathematics teaching, it is important for prospective teachers to see the connection between mathematics and pedagogy. Therefore, teacher educators should make the goals of mathematics methods courses explicit and make the situations prospective teachers encounter both mathematical and pedagogical.

In this study, the discourse represented mathematics and pedagogy as two separate entities instead as a cohesive unit. Sherin (2002) has noted that when considering a topic, teachers tend to think of subject matter knowledge and pedagogical knowledge as an integrated unit. The prospective teachers engaged in their mathematical tasks from the viewpoint of a middle or high school student, but did not engage in the tasks as a teacher teaching the task to students. Therefore, they did not discuss the pedagogical skills necessary to implement the task. They did not make the connection that mathematical explorations were supposed to show examples of pedagogy. Shulman (1986) argued that the key to describing the knowledge base for teachers lies at the intersection of content and pedagogy. Sherin (2002) has reiterated Shulman's point by asserting that pedagogical knowledge acts on subject matter knowledge to produce pedagogical content knowledge. Even and Tirosh (1995) have noted that the influence of subject matter knowledge on pedagogical content choices needs further investigation in mathematics education research.

Finally, teacher educators should present discourse as a method of teaching and learning to prospective teachers. Marsh (2002) advocated explicitly teaching prospective teachers about the concept of discourse and its facilitation in the classroom. Specifically, she advocated teaching prospective teachers the benefits of incorporating multiple methods of discourse. By doing so, prospective teachers will be better able to create communities in which students can discuss and reflect on their mathematical learning (NCTM, 1991, 2000).

Lo and Wheatley (1994) claimed that problematic situations are the result of interacting with others through verbal and written communication. If prospective teachers are going to answer the call of mathematics reform efforts to engage students in the active construction of knowledge through communication, then they should participate in and learn how to facilitate productive discourse during their training. Mathematical discourse occurs in mathematics classes. However, the mathematics methods course provides a forum for prospective teachers to participate in mathematical discourse and then analyze the nature of the discourse and the ways in which the discourse affected knowledge construction. Discourse becomes a central focus of mathematics methods courses. Methods courses have the potential to integrate the *what* of purposeful discourse with the *how* of its effective facilitation.

McNair (2000) advocated the development of explicit mathematics goals as a step toward maximizing learning potential of classroom discussions. Borko et al. (1992) suggested that teacher educators incorporate talk as an objective in methods courses. They referred to talk as discourse about mathematics teaching. By doing so, prospective teachers talk through their reasoning and solution processes with others who might be more proficient in an area of mathematics and can offer another perspective or mathematical representation. Davis and Simmt (2006) suggested that methods courses integrate discourse skills that involve vocabulary, images, and algorithms used in K-12 curriculum. Integrating discourse as a focal point of mathematics methods courses should facilitate the development of prospective teachers who view mathematics in a way that welcomes conversation and collaboration.

Implications for Future Research

Examining the factors that contribute to mathematical self-efficacy is a suggestion for future research. Examining the factors that promote self-efficacy in mathematics and at what point this self-efficacy can begin to diminish can be beneficial to mathematics education research. Bandura (1993) has cited self-efficacy beliefs as the strongest predictor of motivation. If an individual possesses an interest in a task and believes in her own ability, then she can likely accomplish the task. Examining the self-efficacy beliefs of prospective mathematics teachers can provide insight into how they view mathematics. This examination could determine whether prospective mathematics teachers possess an expert blind spot (Nathan & Petrosino, 2003). Possessing an expert blind spot toward mathematics can make learning multiple representations and alternate solution methods difficult for prospective teachers because of their difficulty in understanding *why* a student cannot understand a concept.

The examination of prospective mathematics teachers' self-efficacy beliefs when they encounter a problematic situation is intriguing. This examination can lead to prospective teachers' own development of mathematical self-efficacy along with skills to cultivate mathematical self-efficacy in their own students when they encounter a problematic task. Furthermore, teacher educators could benefit from knowing the motivation factors of prospective teachers. McNair (2000) advocated understanding the intentions that motivate prospective teachers' participation in classroom activities. Understanding prospective teachers' intentions provides teacher educators with insight on their perspective toward the mathematics methods course and the specific tasks they encounter in the course.

Examining prospective teachers' beliefs toward mathematical procedural and conceptual learning is a suggestion for future research. Philippou and Christou (1998) surmised that conceptions, beliefs, and attitudes toward mathematics play a determining role in the development of teaching practices. All of the interview participants stated that they had strong conceptual learning of mathematics. However, their discourse only reflected procedural learning. The prospective teachers had opportunities to reflect their conceptual knowledge of mathematics but did not demonstrate the depth of their conceptual knowledge. A question that requires further examination is whether prospective teachers can distinguish between procedural and conceptual understanding. Philippou and Christou discovered that the prospective teachers in their study professed a conceptual understanding of mathematics, yet only demonstrated a procedural understanding of the mathematical concepts they encountered.

A third suggestion for future research is to examine prospective teachers' expectations toward their mathematics methods training. In this study, the mathematics methods course contained learning goals that integrated pedagogical learning with mathematical content learning. The stated expectations of the prospective teachers who were interviewed did not align with this goal. Borko et al. (1992) found that prospective teachers placed more importance on learning activities that they could use immediately than theoretical or conceptual information. Therefore, an examination of prospective teachers' learning expectations toward teacher development would be beneficial to teacher educators. It is beneficial for teacher educators to be aware of the expectations and assumptions possessed by prospective teachers because they influence the perspective of prospective teachers in their interactions with the content of the course. Simon (1995) has noted that prospective teachers should explicitly state their assumptions, beliefs, and theories about teaching. Borko et al. have asserted that prospective teachers' fundamental beliefs about learning, teaching, and learning to teach should be challenged. Prospective teachers need to be explicitly aware of the purpose of methods courses and the reasons behind these purposes so that they view the course as meaningful.

A final suggestion for future research is to examine prospective teachers' beliefs and perspectives on discourse. Pierson (2008) asserted that many teachers do not realize the benefits of using discourse in the classroom as a tool for learning. In order to know the benefits of mathematical discourse, prospective teachers need to be able to identify quality discourse and then learn ways to facilitate this discourse in the classroom. Prospective teachers need to ponder what productive mathematical discourse looks like and how to teach students to engage in mathematical discourse. By examining prospective teachers' beliefs regarding discourse, teacher educators can examine existing barriers that may inhibit prospective teachers from incorporating mathematical discourse in their classroom such as classroom management, standardized testing pressures, and time constraints. By knowing prospective teachers' beliefs about barriers toward the implementation of classroom discourse as a learning tool, teacher educators can work to eliminate those beliefs that hinder the facilitation of mathematical discourse.

Researcher's Reflection

As I reflected on the entire process of this study, I realized that I have altered my beliefs about mathematics education. From my research, I have discovered elements of mathematics learning that at the outset of this study did not play into my thinking. My assumptions about teacher knowledge and preparation have been proven incorrect. I believe that is the purpose of research. Research should make the reader question assumptions on a topic and allow readers to construct meaning based on the findings and discussions of the study. Conclusions result from the existence of data and not pre-existing assumptions or experiences.

I desired a study that produced relevant findings to a practical situation that can lead to practical implications for teacher education. I feel that this study has yielded relevant findings with respect to an issue that has affected my life in terms of my profession as a school administrator. When I interact with new teachers, I question their perspective on mathematics teaching and learning. The findings of the present study have expanded my interactions with mathematics teachers because I look at what students say and the ways in which they talk during instruction differently and more critically. The implications discussed from this study have the ability to alter the preparation of future prospective teachers.

I did not know much about classroom discourse and more specifically, mathematical discourse at the beginning of this study. I had held the belief that discourse was simply conversations people had. However, this study has proven to me that classroom discourse is and should be, so much more. Classroom discourse is about the potential for shared knowledge. It is about different thoughts and ideas meeting at an intersection and collectively, learners decide which path to take. Mathematical discourse is a *natural* method for teaching and learning. Through mathematical communication, students and teachers learn new mathematical perspectives and representations that can alter the knowledge one has of a mathematical concept. Mathematical discourse is a powerful and effective tool that is overlooked in the mathematics classroom.

I believe that the participants of this study spoke freely and openly about what they considered important to their knowledge base. I captured their voices and ideas to the best of my ability. It was not my intention, nor should it be the perspective assumed by any readers, that I depicted the nature of their conversations as wrong or incorrect. Discourse is not about right and wrong. Discourse takes the shape of what is interesting and relevant to the participant. The fact that the participants yearned for pedagogical and operational knowledge more than mathematical knowledge is relevant to the practice of teacher preparation. The fact that the prospective teachers did not engage in mathematical discourse should not be a reflection on the prospective teachers or the professor. It should reflect the need for explicit focus and instruction about mathematical discourse.

This study has made me grow as a person and as a learner. The arduous dissertation journey has enlightened me to characteristics that I thought I lacked and thought I could never have. I view mathematics teaching in a different light, with a different, more cohesive perspective. From this study, I have expanded my role as a school administrator to an instructional inquirer. I advocate for the integration of mathematical discourse in the instruction of the mathematics teachers with whom I work.

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Appendix A: MAT Program Description and Methods Course Description

PROGRAM DESCRIPTION & COURSE OUTLINE
MASTER OF ARTS IN TEACHING IN
SECONDARY MATHEMATICS

[Large University's] Master of Arts in Teaching (M.A.T.) is for individuals who already possess a bachelor's degree in mathematics and who are interested in secondary (6-12) certification in mathematics. The Master of Arts degree program leads to initial certification of well-qualified teacher candidates and prepares them to be teacher-leaders.

The M.A.T. program is a content-focused, standards-based program, which meets the Professional Standards Commission standards for the degree. The program addresses the Board of Regents Principle #5: Teacher preparation programs will be the shared responsibility of education faculty, arts and sciences faculty, and classroom teachers in the schools. and Principle # 8: The University System will encourage the institutions to develop new and innovative teacher preparation programs to respond to state need and to contribute to increased student learning and achievement in public schools.

Course work emphasizes scholarly rigor through research and engagement in a variety of field-based action research projects. Technology and multicultural considerations are infused throughout the program. This Masters' program leads to initial certification and so is not appropriate for teachers who are already certified.

PROFESSIONAL SEQUENCE (30 hours)			
COURSE	**Hours**	**Semester**	
EDUC 6100: Development, Psychology, and Diversity of the Learner	5	SP/SU	
EDUC 6100L: Practicum I	1	SP/SU	
EDUC 6200: Curriculum, Assessment, and Classroom Management	3	SU/FA	
MAED 6416: Teaching Secondary Mathematics I	3	FA	
MAED 6416L: Practicum II	3	FA	
MAED 6475: Teaching Secondary Mathematics II	3	SP	
MAED 6475L: Practicum III	6	SP	
EDUC 6300: Reflective Inquiry and Action Research	3	SP	
EDUC 6400: Portfolio	3	SU	
TEACHING FIELD (18 hours)			
MATH 7700: Elementary Set Theory	3	SU	
MATH 7712: Discrete Mathematics	3	SU	
MATH 7713: Statistics and Data Analysis	3	SU	
MATH 7714: Geometry from Multiple Perspectives	3	FA	
MATH 7717: Elementary Number Theory	3	SP	
MATH 7718: Functions and Analytic Techniques	3	SU	
TOTAL	48		

PROGRAM TOTAL: 48 SEMESTER HOURS

[Large University]. (2010). 2010–2011 *Course catalog*.

COURSE DESCRIPTION

MAED 6416 - Teaching Secondary Mathematics I

MAED 6416 - Teaching Secondary Mathematics I

3 Class Hours 0 Laboratory Hours 3 Credit Hours

Prerequisite: EDUC 6100 and EDUC 6100L.

An examination and application of curriculum issues, learning theories, teaching strategies, instructional materials and assessment procedures for teaching middle and secondary school mathematics in the multicultural and diverse classroom of today. Includes a secondary school field experience in mathematics teaching and seminars.

Note Proof of professional liability insurance is required prior to school placement.

[Large University]. (2010). 2010–2011 *Course catalog*.

Appendix B: Consent Forms

CONSENT FORM

I agree to participate in the research project entitled **An Examination of the Discourse within a Mathematics Methods Course** that is being conducted by Rako Morrissey. I understand that this participation is voluntary; I can withdraw my consent at any time and have the results of the participation returned to me, removed from the experimental records, or destroyed.

The following points have been explained to me:

1.The reason for the research is to examine the factors that affect the discourse of students in a mathematics methods course. I believe examining the factors that affect and influence discourse patterns can have a significant impact on the tailoring of the class for the intended individuals and the benefits that I may expect from it are: more self-awareness to the mathematical beliefs of the participants and how those beliefs transcend into their classroom participation and their mathematical teaching. Through this, teachers become aware of their behaviors as students and how they can be quite influential on their instructional practice as teachers. Additionally, the professor of the class can modify and tailor the instruction to target specific outcomes and learning goals so that instruction is relevant and meaningful to the target audience in their preparation of entering the field of education.

2. The procedures are as follows: The interview will be conducted by the researcher. The researcher will ask the participants a series of questions that address the formation of mathematical beliefs and their view of how students learn mathematics. The interview will be audiotaped for transcription purposes.

3.The discomforts or stresses that may be faced during this research are: psychological stress from participating in the interview process.

4.Participation entails the following risks: psychological stress from participating in the interview process.

5.The results of this participation will be anonymous and will not be released in any individually identifiable form without the prior consent of the participant unless required by law. Audio tapes of the interview will be used for transcription purposes only and will be locked in a desk drawer for one year after completion of the study and then will be destroyed.

____

Signature of Investigator, Date

Signature of Participant, Date

PLEASE SIGN BOTH COPIES, KEEP ONE AND RETURN THE OTHER TO THE INVESTIGATOR

INFORMED CONSENT FORM

I agree to participate in the research project entitled **The Examination of the Mathematical Discourse among Pre-Service Teachers within a Mathematics Methods Course** that is being conducted by Rako Morrissey. I understand that this participation is voluntary; I can withdraw my consent at any time and have the results of the participation returned to me, removed from the experimental records, or destroyed.

The following points have been explained to me:

1. The reason for the research is to examine the mathematical discourse among pre-service teachers within a mathematics methods course. I believe examining the nature, patterns, and functions of classroom discussions can have a significant impact on the tailoring of the class for the intended individuals as well as information regarding the preparation of pre-service teachers in fostering mathematical discourse within the K-12 setting. The benefits that I may expect from it are: more self-awareness of the mathematical discussions of the participants and how these discussions transcend and support the facilitation of mathematical discussions in their classroom instruction. Through this, teachers become aware of the power of mathematical discourse and how it can be quite influential on their instructional practice. Additionally, the professors who teach preservice teachers can examine the discourse that occurs in their instruction so that the activities and topics within instruction are relevant and meaningful to pre-service teachers' development of mathematical discussion skills in their preparation of entering the field of education.

2. The procedures are as follows: Classroom discussions will be audiotaped without researcher interaction or participation. Documents will be collected from the class. Participants will complete a two question survey within 24 hours of the class. The survey responses will be completed utilizing Survey Monkey and will be anonymous. No IP addresses will be collected and has been noted on the Survey Monkey registration.

3. The discomforts or stresses that may be faced during this research are: Any minimal stress that would incur would be associated with the discomfort of classroom discussions being audiotaped.

4. Participation entails the following risks: There are no known risks involved in participating in this research.

5. The results of this participation will be anonymous and will not be released in any individually identifiable form without the prior consent of the participant unless required by law. Participants of the interview will not be required or asked to place name on the interview.

____

Signature of Investigator, Date

Signature of Participant, Date

PLEASE SIGN BOTH COPIES, KEEP ONE AND RETURN THE OTHER TO THE INVESTIGATOR

Appendix C: Classroom Observation Protocol

Classroom Observation Protocol

Date ____________________________

Task Discussion is Centered Around	Type of Discourse	Type of Teacher Knowledge	Category of Communication	Description of Questions/Conversation Key Observations

Appendix D: Interview Protocol

Mathematical Discourse Interview Protocol

1. What did you feel was the purpose of the mathematics methods course?

2. Did you feel the focus of the class was more on math content, how to teach math as teacher, or on operational issues?

3. What information did you feel you needed from the class to help develop you as a mathematics teacher? Did you receive that information in the class?

4. Is there any knowledge that you were hoping to gain and did not?

5. What do you feel is essential knowledge for you as a pre-service teacher?

6. How important do you feel talking about mathematics is for your own development and for student learning in your classroom?

Mathematical Discourse Interview Protocol Follow-Up Questions

1. Describe the extent to which you think you have a conceptual understanding of mathematics.

2. When I analyzed the data from my observations, I noticed that the students, in general, spoke in class more often when the discussion was about running a classroom than they did when talking about mathematical tasks. Why do you think that is?

3. Could the lack of discussion have stemmed from knowing more procedural knowledge of math concepts than conceptual knowledge and therefore not being comfortable in discussing the concepts?

4. Could the lack of discussion about mathematics have been from disengagement? Since you already know the concepts, do you think your group would have rather focused on something else?

Appendix E: Reflective Writing Prompt

Reflective Writing Prompt

How did you help make this discussion better?

Select each one that applies to you. Choose one to reflect about in a small narrative below.

_____ I learned a math idea from someone based on our discussion.

_____ I taught someone a math idea through my conversation and experiences.

_____ I looked at a math idea in a new way based on our discussion.

_____ I disagreed or questioned an idea in our discussion today.

_____ I referred to someone else’s idea from research.

Appendix F: Data Analysis Task

Name ______________________________ Period ____________________

Student Name	Word Recall	Word Recall With Objects
3	guitar	ship 6
9	piano	baby 10
6	horse	pony 10
10	ship	paperwad 14
10	purse	Feather 12
10	basketball	fork 15
7	turkey	hair 7

1. Find the Mode for both sets of data.

   Word Recall Mode ____10____ Word Recall w/Objects Mode ____10____

2. Find the Range of both sets of data.

   Word Recall ~~Range~~ 9 Word Recall w/Objects ~~Range~~ Median 10

   Median

   6, 7, 10, 10, 12, 14, 15

   3, 6, 7, 9, 10, 10 10

3. Find the Mean for both sets of data.

   Word Recall Mean Word Recall w/Objects Mean 10.571

   7.857

4. If you were trying to convince a group of math teachers that using manipulatives increases learning greatly, what kind of graph would you use to organize this data? Would you use a line graph, bar graph, or circle graph? Construct the appropriate graph below.

Construct a Box and Whiskers Plot for both sets of data

STEPS:

1) Arrange data in order from least to greatest.
2)Draw a line with divisions that count by 2's, 5's, 10's, 20's, etc. whatever is appropriate.
3) You will make 5 dots: -
   - one at the median
   - one at the upper quartile (the median of the upper half)
   - one at the lower quartile (the median of the lower half)
   - one at the greatest number
   - one at the lowest number

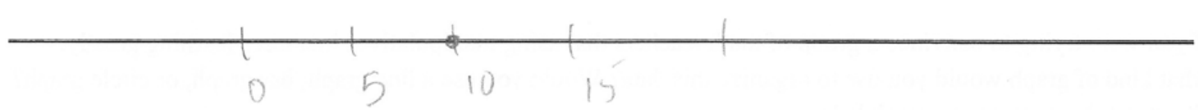
4) Draw a box around the 3 middle dots.
5) Draw whiskers (straight horizontal lines) from the box to each end dot.
6) Determine the outliers, if any.

HOW TO DETERMINE THE OUTLIERS

Step 1: Determine the interquartile range (subtract the lower quartile from the upper quartile)
Step 2: Multiply that by 1.5.
Step 3: Add that number to the upper quartile. Any number greater than this number is considered an outlier (an extreme).
Step 4: Subtract the number that you got when you multiplied by 1.5 from the lower quartile. Any number less than this number is considered an outlier.

If you have outliers, the whisker cannot be connected to them! Erase the whisker back until the end of the whisker lies at the next data back from the outlier.

25% ≤ 6
50% ≤ 9
75% ≤ 10
100% ≤ 10



Appendix G: Unit Circle and Trigonometry Task

Various Topics in Trigonometry

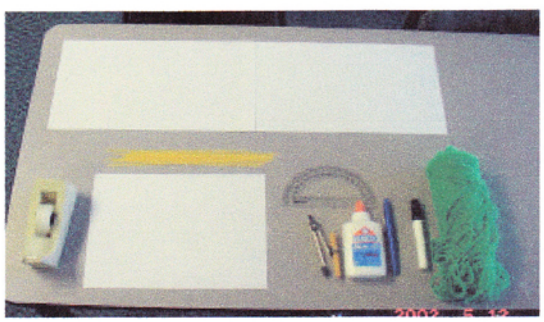
<u>Lesson 1 – Spaghetti Trig – Developing the graphs of sine and cosine functions from the unit circle</u>

Objectives:

Students will develop a better understanding of the unit circle.
Students will make the connection between the unit circle and the graphs of the sine and cosine trig functions.

Materials Needed:

Poster board or two legal sheets for each student
Yarn (a light color preferably)
Tape or Glue
Protractors
Compasses
Spaghetti
Markers (a different color than the yarn)
One sheet of letter-size paper for each student



Prep before class begins:

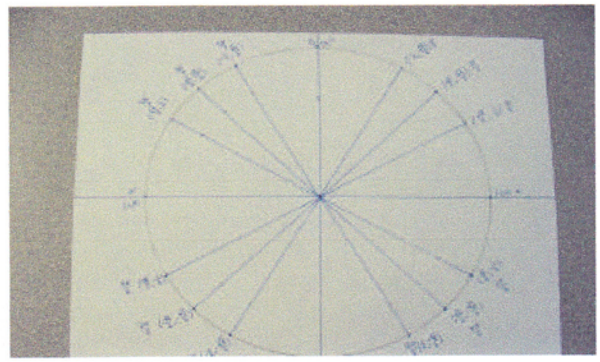
You will need 50-65 minutes to complete the project, so the more you have prepared for the students, the better off you will be. The students will need the following items at their desks before class begins. They will need white poster board or two legal sheets taped together to make one long 8 ½" x 23" sheet, yarn (at least 3 feet for each student), tape or glue, protractor, compass, 7-10 noodles of spaghetti, and a marker.

Procedure:

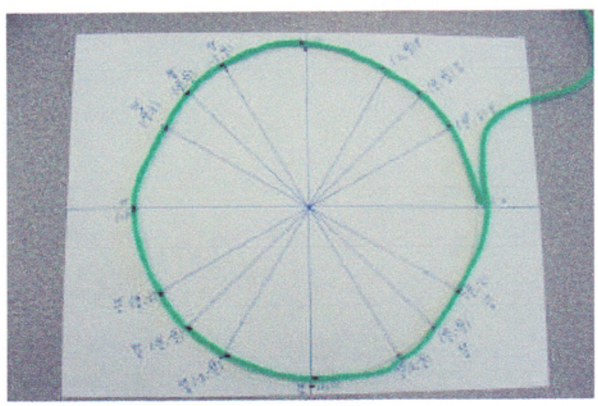
The first part of this lesson could actually be done on a separate day. Students should create a unit circle on the letter-sized paper. Begin by having the students draw an x and y axis on the letter-sized paper. The origin should be as close to the center of the paper as possible. Use the compass to create a circle with a center at the origin and a radius of at

Sturdivant, M. (2002). *Various topic in trigonometry: Lesson 1–spaghetti trig.*

least 3 ½". Use the protractor to measure angles of $\pi/6$, $\pi/4$, and $\pi/3$. The point where the terminating side of these angles intersect with the unit circle will have ordered pairs of $(\frac{\sqrt{3}}{2}, \frac{1}{2})$, $(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$, $(\frac{1}{2}, \frac{\sqrt{3}}{2})$ respectively. Then have students measure the same reference angles in each of the remaining quadrants, marking the coordinates of the ordered pairs as they go (paying careful attention to sign). The students should also mark the ordered pairs for the points where the unit circle intersects the x and y axes as well.



Now have the students wrap the string around the unit circle. One end of the string should be placed at (1, 0). Use the marker to mark points on the string at each of the special angles.

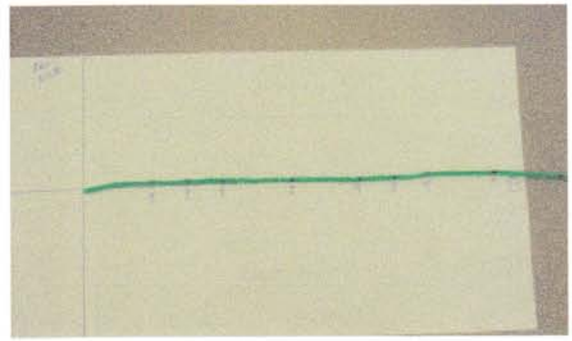


It is important to note to the students at this point that we are interested in finding a function in terms of the angle θ that will give us the value of the x coordinate and a function in terms of the angle θ that will give us the value of the y coordinate. From previous study, the students should know that the functions we are looking for are sine and cosine.

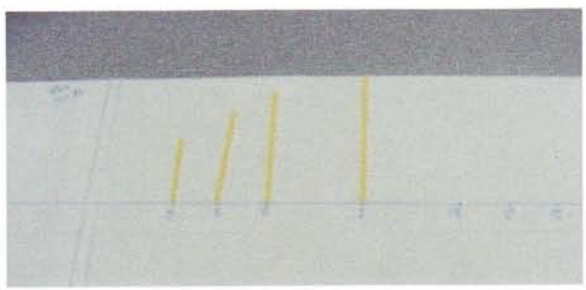
Remind the students that $x = \cos(\theta) = f(\theta)$ and $y = \sin(\theta) = g(\theta)$. This means that if we are to graph the sine function our new independent variable is the angle θ. The remainder of this lesson plan will discuss completing the project for graphing the sine function. It is easily adapted to graphing the cosine function.

Have the students draw a coordinate axis on their poster board or legal paper. Since θ is the independent variable, the horizontal axis should be labeled θ and the vertical axis should be labeled $g(\theta) = \sin\theta$.

The yarn can now be placed along the horizontal axis with the end of the string that started at the point (1,0) now placed at the origin since this represents $\theta = 0$. The marks that were made on the yarn will give us the angle measures that we need. Thus the first mark can be transferred to the poster board and labeled $\pi/6$. The second mark can be transferred to the poster board and labeled $\pi/4$, etc. Once the students have labeled the positive horizontal axis, they can flip the string over and label the negative horizontal axis using the same marks. Note to the students that this is using negative angle measures.

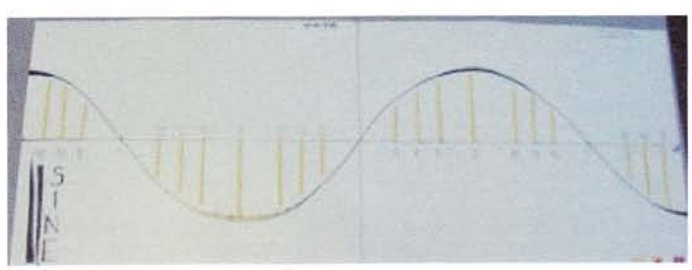


We will now go back to our unit circle. Since we are interested in the sine function which corresponds to the y value, we are interested in the distance a given point is from the horizontal axis. This is where the spaghetti will help. At an angle measure of 0, the y value is 0, thus we will not need any spaghetti. However, at $\pi/6$, the y value is ½. Have the students place a noodle of spaghetti vertically with one end on the horizontal axis and the remainder of the noodle going through the point ($\frac{\sqrt{3}}{2}$, ½). They should break the noodle off at that ordered pair. Thus the noodle gives us a visual image of the y value (sin θ) at $\pi/6$. Take that piece of spaghetti and glue it on the poster board at $\pi/6$. Be sure to tell the students that since the y value was positive, then the spaghetti should be placed above the horizontal axis. Repeat this process at each special angle. When the students get to the third and fourth quadrants, remind them that they should now be placing their spaghetti below the horizontal axis.



Be sure to have the students also work with the negative angles as well, reminding them that $-\pi/6$ will be in the 4th quadrant and so the spaghetti will be placed below the horizontal axis. When the students finish they should see a perfect sine curve. You can fill in the gaps by having the students measure angles of $\pi/12$ and $5\pi/12$ and their corresponding reference angles around the unit circle.

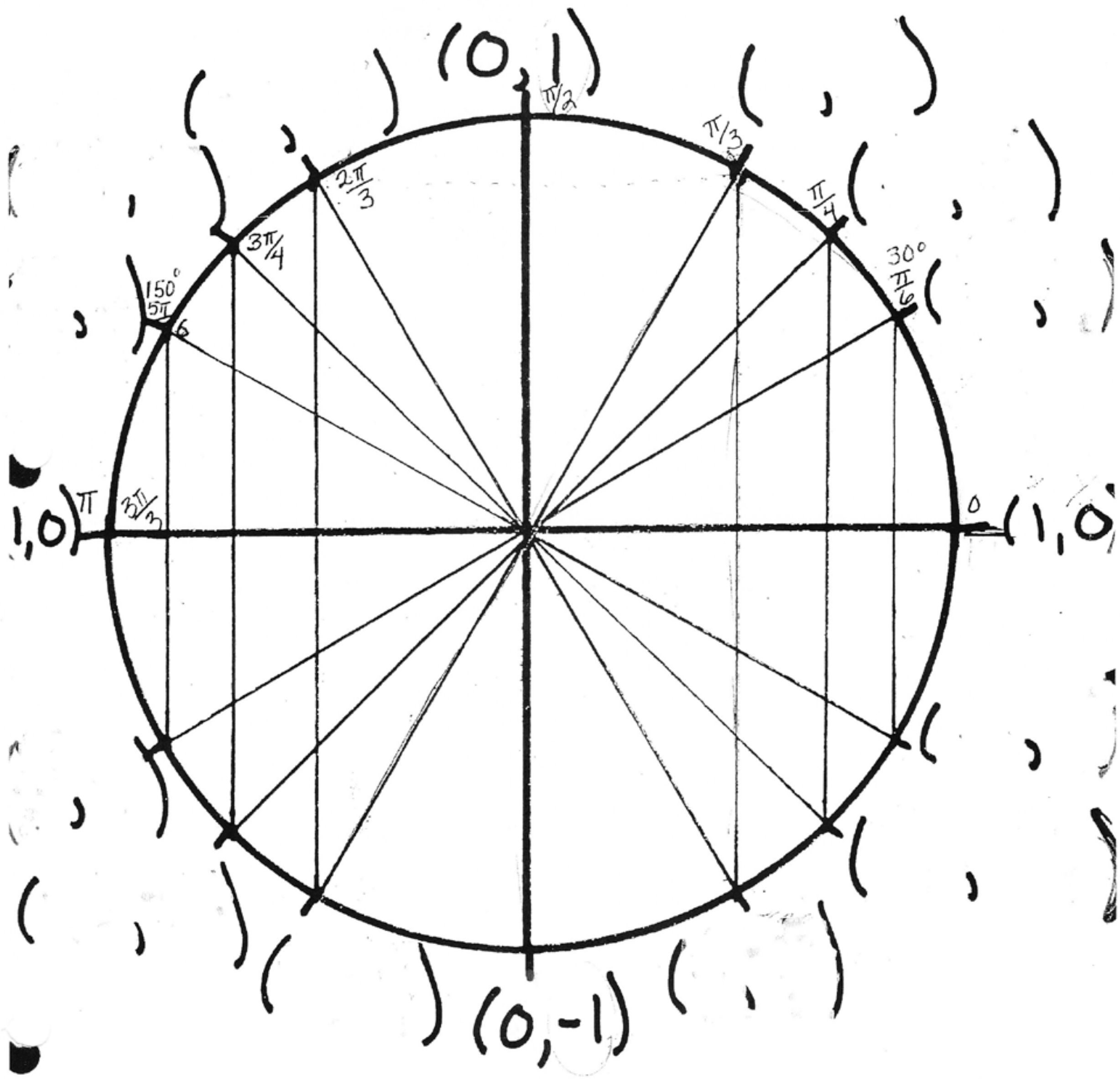
It may be a good idea to have half of your students graph the sine function and half graph the cosine function. The activity is identical with the exception that the spaghetti will be used to measure the ordered pairs distance from the vertical axis in the unit circle.



Sturdivant, M. (2002). *Various topic in trigonometry: Lesson 1–spaghetti trig.*

Unit Circle Values

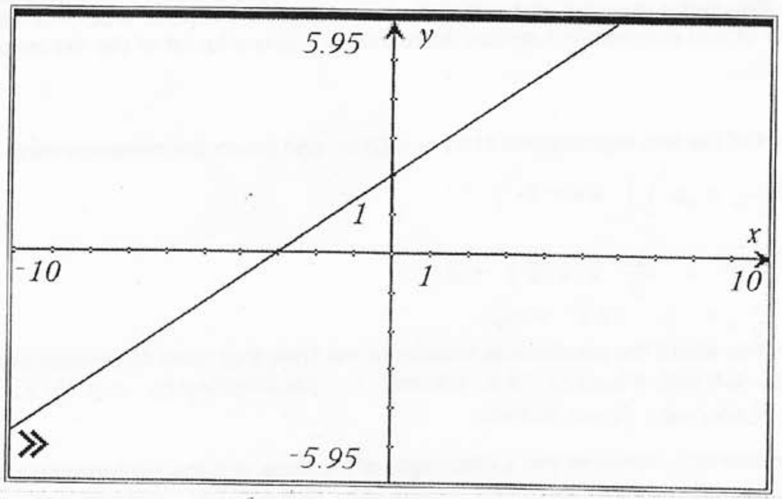
$(x, y) \Leftrightarrow (\cos\theta, \sin\theta)$



Appendix H: Polynomial Task

BUILDING POLYNOMIAL FUNCTIONS

1. What is the equation of the following linear function?



f1(x) = $\frac{2}{3}x + 2$

2. How did you find it? found slope, y-intercept

3. Graph the function on your handheld. Use a square window.

4. The slope/y-intercept form of a linear function, $y = mx + b$, is commonly used to find the equation of the function. If your function is not in this form, put it in this form.

5. Rewrite the linear function by factoring out the slope, m, that is, let $y = m(x + b/m)$.

$y = \frac{2}{3}(x + 3)$

6. Choose a different linear function written in the form $y = mx + b$, where $m \neq 0$. Write it below and graph it on your handheld.

f2(x) = $4x + 6$

Rewrite this equation in the form $y = m(x + b/m)$. $y = 4(x + \frac{3}{2})$

7. For each function, what does $-b/m$ represent, that is, what is the connection between these values and the graph of each linear function? - Xintercept-

Letting $c = -b/m$, the form $y = m(x - c)$ might be called the *slope/x-intercept* form of the equation, where c is the x-intercept of the line. The factor theorem states that if c is a root (x-intercept) of a polynomial function, then $(x - c)$ must be a factor of that polynomial function. Note that $(x - c)$ is a factor of the expression. The only other factor is the slope, m.

8. Find the product of the two expressions f1(x) and f2(x), and graph the corresponding quadratic function.

$$\left(\frac{2}{3}x + 2\right)(4x + 6)$$

$$= \frac{8}{3}x^2 + \frac{12}{3}x + 8x + 2$$

$$= \frac{8}{3}x^2 + 12x + 12$$

9. What do you notice about the parabola in relation to the lines that were its components?
The parabola shares the same x-intercepts as the two linear functions

10. Describe the relationship between the x-intercepts of the lines and the x-intercept(s) of the parabola.
The x-intercepts of the lines are the roots of the parabola

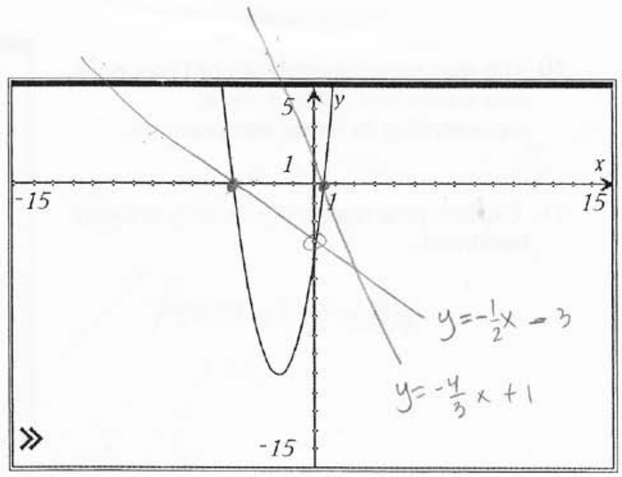
11. Describe the relationship between the y-intercepts of the lines and the y-intercept(s) of the parabola.

linear both positive → parabola positive
linear both negative → parabola positive

12. Find the value halfway between the two x-intercepts (the average x-value) of the linear equations. What do you notice about this value in relation to the parabola.

axis of symmetry

WORKING BACKWARD



1. Given the parabola at the right, sketch lines that could represent its linear factors.

2. Did you use both the x-intercepts and the y-intercept of the parabola to make your choices? Explain.

3. What else did you consider while making your choices?

$\left(-\frac{1}{2}x-3\right)\left(-\frac{4}{3}x+1\right)$

$\frac{4}{6}x^2-\frac{1}{2}x+\frac{12}{3}x-3$ (4)

$\frac{2}{3}x^2+\frac{7}{2}x-3$

4. Write linear equations for these lines you chose.

$y=-\frac{1}{2}x-3$

$y=-\frac{4}{3}x+1$

5. Given these linear equations, what would be the equation of the parabola? Write in the form $y = ax^2 + bx + c$

$\frac{2}{3}x^2+\frac{7}{2}x-3$

6. On a new page, graph all of these equations on your handheld. Adjust the viewing window if necessary.

✓

7. Would another pair of lines work? If so, show another set of lines on your handheld.

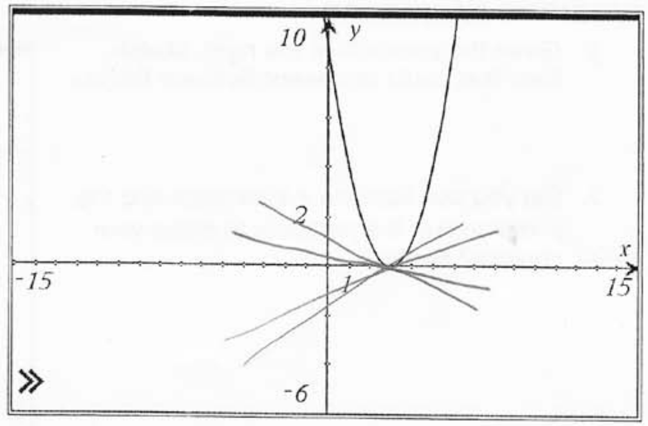
many

8. Given this new set of linear equations, what would be the equation of the parabola?

The same

9. What do you notice regarding the product of the linear factors? Same roots.

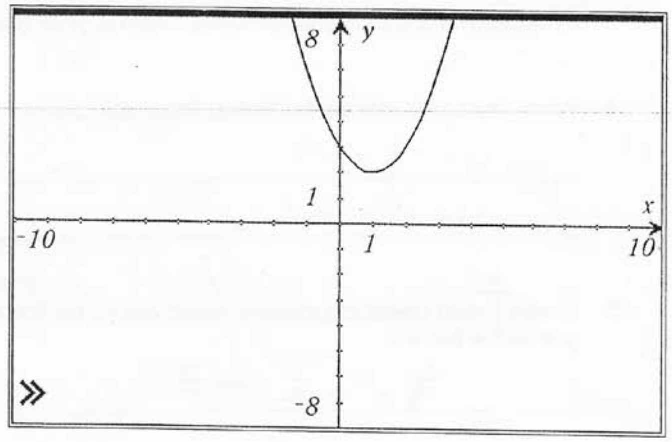
10. The quadratic function at right has only one x-intercept. Sketch lines representing its linear components.



11. Explain your reasoning. Verify on your handheld.

one solution

12. Try to sketch lines that are components of the quadratic function at right.



13. What did you find?

14. What is the reason for what you found?

no solution

Appendix I: Surface Area and Volume Task

Boxes

Background: In the packaging industry, we have a variety of boxes of many dimensions to choose from. We need the variety in order to accommodate the multitude of products which might be shipped. Since cardboard costs money, it is important to know which box will best fit a product in order to minimize costs.

Group arrangement: Students will work in pairs but each student will have an individual computer

Procedure: For the first table, follow the directions under "Changing one dimension".

Changing One Dimension

	Length	Width	Height	Surface Area	Volume
1	2.0 cm	2.0 cm	2.0 cm	24 cm^2	8.02 cm^3
2	2.0 cm	4.0 cm	2.0 cm	41 cm^2	16.37 cm^3
3	4.0 cm	4.0 cm	2.0 cm	64 cm^2	31.67 cm^3
4	4.0 cm	4.0 cm	4.0 cm	96 cm^2	63.77 cm^3

Divide the volume of each prism by the volume of prism 1. Reduce each fraction to a ratio over 1.

$\frac{\text{Volume 2}}{\text{Volume 1}} = \frac{16.37}{8.02} = \frac{2.04}{1}$ $\frac{\text{Volume 3}}{\text{Volume 1}} = \frac{31.67}{8.02} = \frac{3.9489}{1}$ $\frac{\text{Volume 4}}{\text{Volume 1}} = \frac{63.77}{8.02} = \frac{7.95}{1}$

Divide the surface area of each prism by the surface area of prism 1. Reduce each fraction to a ratio over 1.

$\frac{\text{SA 2}}{\text{SA 1}} = \frac{41}{24} = \frac{1.708}{1}$ $\frac{\text{SA 3}}{\text{SA 1}} = \frac{64}{24} = \frac{2.66}{1}$ $\frac{\text{SA 4}}{\text{SA 1}} = \frac{96}{24} = \frac{4}{1}$

How did you change a dimension each time? What did you notice happened to your volume when you changed that dimension? Was the effect on surface area as great? From 1 to 2, we changed the length & width, From 2 to 3, changed length & Height, 3-4 changed height. 1-2 it doubled. 1-3 x 4, 1-4 x 8. No.

How can you explain your results above? A change in one dimension may not necessarily affect a change in the Surface Area as much as the Volume.

Procedure: Follow the directions under "Improving Volume"

Improving Volume

	Length	Width	Height	Surface Area	Volume
1	3.8	2.0 cm	3.4	54 cm^2	25.60
2	3.1	3.0 cm	2.9	54 cm^2	27.18
3		4.0 cm		54 cm^2	
4		2.0 cm		96 cm^2	
5		4.0 cm		96 cm^2	
6		6.0 cm		96 cm^2	
7		3.0 cm		150 cm^2	
8		5.0 cm		150 cm^2	
9		7.0 cm		150 cm^2	

You tried to get the biggest volume given a specific surface area. What were the dimensions that gave you the greatest volume for your three surface areas?

54 cm^2 length:_________ width: _________ height: _________
96 cm^2 length:_________ width: _________ height: _________
150 cm^2 length:_________ width: _________ height: _________

What similarities do you notice about the dimensions for each of these prisms?

L/W should be about even

Appendix J: Probability Task

Guided Notes- Probability

1. Fill out the KWL chart regarding probability

Know	Want to Know	Learned

2. What are some ways to represent a probability? ratio, fractions, decimals 0-100%
3. What is a sample space? set of all possible outcomes
4. What is a sample space of a coin? {heads, tails}
5. What is the sample space for a die? {1, 2, 3, 4, 5, 6}
6. How do you find experimental probability? $= \frac{\text{\# times event occurs}}{\text{\# trials}}$
7. How do you find theoretical probability? $= \frac{\text{\# ways event can occur}}{\text{total \# equally likely outcomes}}$
8. Fill out the remainder of the chart!

Suit	Number of Times Pulled in experiment	Experimental probability of pulling this suit	Theoretical Probability of pulling this suit
Diamonds	3	$3/20$	$1/4$
Hearts	4	$4/20 = 1/5$	$1/4$
Spades	12	$12/20 = 3/5$	$1/4$
Clubs	1	$1/20$	$1/4$
	Total=20		

Tossing a Coin:

1. What is the sample space for a coin? {heads, tails}

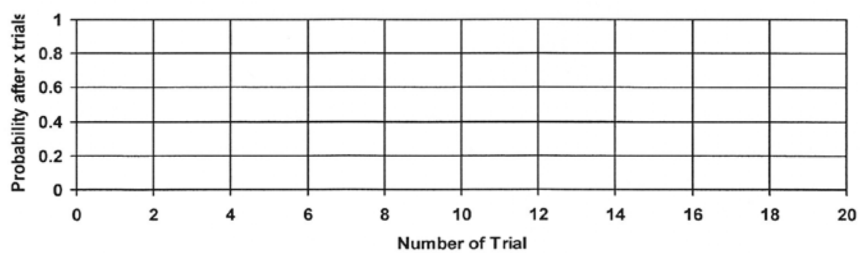
2. Toss the coin 20 times and put an x in the chart when a heads is tossed. Then fill out the cumulative number of heads in the 3rd row.

Trial #	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Heads?	✓	X		X	X	X	X			X				X	X			X		
Total # of heads so far	1	2	2	3	4	5	6	6	6	7	7	7	7	8	9	9	9	10	10	10.

3. Find the experimental probability and theoretical probability after every 2 trials.

Number of trails so far	Total number of heads so far	Experimental probability	Theoretical Probability
2	2	2/2 = 100%	1/2
4	3	3/4 = 75%	1/2
6	5	5/6 = 83%	1/2
8	6	6/8 = 75%	1/2
10	7	7/10 = 70%	1/2
12	7	7/12 = 58%	1/2
14	8	8/14 = 57%	1/2
16	9	9/16 = 56%	1/2
18	10	10/18 = 55%	1/2
20	10	10/20 = 50%	1/2

4. Graph both the Theoretical and Experimental probabilities below. Think about what makes a graph continuous and what makes a graph discrete. Use different colors for each type of probability and label your graph.



5. What do you notice about the relationship between theoretical and experimental probability as the number of trials increases?
experimental prob. decreasing, theoretical always = 50% (Sample space contain 2 outcomes)

6. What do you think would happen to the experimental probability if we tossed the coin 200 times instead of 20 times?
experimental probability → 50%

Roling a die

1. What is the sample space for one die? {1, 2, 3, 4, 5, 6}

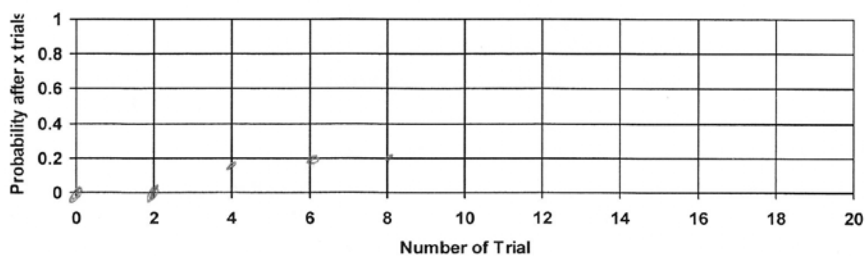
2. Roll the die 20 times and put an x in the chart when a 6 is tossed. Then fill out the cumulative number of 6s in the 3rd row.

Trial #	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
~~Heads~~? 6						X														
Total # of 6s so far	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

3. Find the experimental probability and theoretical probability after every 2 trials.

Number of trails so far	Total number of 6s so far	Experimental probability	Theoretical Probability
2	0	0	1/6 = 17%
4	0	0	
6	1	1/6 – 17%	
8	1	1/8 13%	
10	1	1/10 10%	
12	1	1/12 8%	
14	1	1/14 7%	
16	1	1/16 6%	
18	1	1/18 6%	
20	1	1/20 5%	

4. Graph both the Theoretical and Experimental probabilities below. Think about what makes a graph continuous and what makes a graph discrete. Use different colors for each type of probability and label your graph.



5. What do you notice about the relationship between theoretical and experimental probability as the number of trials increases?
Theoretical always 17%, expermental ↓

6. What do you think would happen to the experimental probability if we rolled the die 200 times instead of 20 times?
same

Appendix K: Factoring Task

Factoring Frenzy!!!

Tic-Tac-But No Toe

Part 1: In the following tic tac's there are four numbers. Find the relationship that the two numbers on the right have with the two numbers on the left.

-90	10
1	-9

36	-6
-12	-6

-36	-6
0	6

-30	-6
-1	5

-49	7
0	-7

120	30
34	4

-81	9
0	-9

-24	-6
-10	-4

-72	24
21	-3

16	4
8	4

-6	-3
-1	2

49	-7
-14	-7

Observations

1. What did you find?
2. Did it follow the pattern every time?

Factoring Frenzy!!!

Tic-Tac-But No Toe

Part 2: Use your discoveries from Part 1 to complete the following Tic Tac's.

9	9
10	1

16	-8
-10	-2

18	6
9	3

6	6
7	1

-35	-7
2	-5

4	-4
-5	-1

45	9
14	5

6	-3
-5	-2

-3	-3
-2	1

-15	5
2	-3

72	-36
-38	-2

-6	-6
-5	1

-72	8
-1	-9

-36	+9
5	-4

-22	11
9	-2

Observations

3. Did your discovery work in every case?
4. Can you give any explanation for this?

Factoring Exploration

The product of two binomials results in a four-term expressions that can sometimes be simplified to a trinomial.

Find the product of the two binomials:

$$(x+3)(x+2)$$

The product of the two binomials is:

$$\boxed{1}x^2+\boxed{5}x+\boxed{6}$$

What is the relationship between the value of "c" and the two constant terms of the binomials?

when multiplied together they give you c

What is the relationship between the value of "b" and the two constant terms of the binomials?

when added together they give you b

To factor the trinomial, reverse the above process.

The process of factoring takes a trinomial and turns it into the product of two binomials.

Consider the given trinomial:

$$x^2 + 6x + 8$$

First, find two numbers that have a product of 8 and also have a sum of 6.

Those two numbers are 4 and 2.

Now create two binomials using these two numbers.

$$(x + \boxed{4})(x + \boxed{2})$$

Does the order in which we place the numbers matter? In other words does the order in which you multiply expressions matter? Explain your answer. depends on the signs of b and c

Use the above method to factor the following trinomials:

1. $x^2 - 6x - 16$ (x−8)(x+2)
2. $x^2 + 7x + 12$ (x+4) (x+3)
3. $x^2 - 3x - 18$ (x+6)(x-3)
4. $x^2 - 7x + 10$ (x−5)(x+2)
5. $x^2 - x - 42$ (x-7)(+6)

Part 2:

What do you expect to happen if there was no “b” term? Explain your answer.

Consider the following trinomial:

$$x^2 - 25$$

First, find two numbers that have a product of __
and also have a sum of __.

Those two numbers are ___ and ___ .
What would be the factorization of the above trinomial?

Use the above method to factor the following trinomials:

6. $x^2 - 36$
7. $x^2 - 81$
8. $x^2 - 4$
9. $x^2 - 9$
10. $x^2 - 100$

Part 3:
Again, consider the following trinomial:

$$x^2 + 10x + 25$$

Find two numbers that have a product of __,
and also have a sum of __.
What do you notice about these two numbers? Explain.

The two binomials that represent the factored form of this trinomial is:

$$(x+\square)(x+\square)$$

What do you notice about the two factors above?

Based on your answer, is there any way to condense the factorization of the trinomial?
If so, why?

Write the condensed factored form of the trinomial:

$$(x+\square)^{\square}$$

Use the above method to factor the following trinomials:

11. $x^2+8x+16$
12. x^2-6x+9
13. $x^2+12x+36$
14. $x^2-14x+49$
15. x^2+2x+1